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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: PDE Books, Present and Future

Theory and Applications of Partial Differential Equations. *By P. Bassanini and A. R. Elcrat.* Plenum Press, New York, 1997. $138.00. 438 pp., hardcover. ISBN 0-306-45640-0.

Partial Differential Equations. Second Edition. *By J. Kevorkian*. Springer-Verlag, New York, 2000. $59.95. 648 pp., hardcover. ISBN 0-387-98605-7.

Applied Partial Differential Equations. *By J. Ockendon, S. Howison, A. Lacey, and A. Movchan*. Oxford University Press, Oxford, UK, 1999. $45.00. xi+427 pp., softcover. ISBN 0-19-853243-1.

Beginning Partial Differential Equations. *By Peter V. O'Neil.* Wiley-Interscience, New York, 1999. $79.95. 512 pp., hardcover. ISBN 0-471-23887-2.

The Past. When I was a post-doc at the University of Arizona in the 1970s, I was chatting one sunny afternoon before colloquium with the late Ruel V. Churchill, who had retired in Tucson and often showed up for mathematics department events. Over coffee and cookies the conversation turned to textbooks. I asked him directly how many copies of his three well-known books, on complex variables, on operational mathematics, and on boundary value problems, had been sold. His answer, astronomical for advanced mathematics texts, shocked me. This was not Michael Crichton! And, in my many retellings of this story to various individuals, I have encountered uniform disbelief from almost everyone, including publishers and local book company representatives. I hesitate to expose this number in a formal review, and therefore it will have to remain an intriguing mystery. However, it may not be so unbelievable when one considers the fact that Churchill's books had been available for over 30 years and were the textbooks of choice for generations of students majoring in mathematics, science, and engineering. One of those texts (see [5]), the one on Fourier series and boundary value problems and now coauthored by J. Brown [4], remains popular, and a sixth edition is in the making.

Which raises the subject of this review.

Most faculty who teach partial differential equations (PDEs) have bookshelves full of elementary PDE books, many formed in the same mold as Churchill's. These look-alikes cover the same topics, namely, Fourier series and integrals, separation of variables, boundary value problems, special functions, and applications to heat flow and vibrations. Some have expanded to several hundred pages, perhaps to ensure that every topic desired by a potential user is included. The market for these texts is the standard undergraduate course in the said topic, usually of one semester. Although

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engineering courses covering this material have cut into this market, the mathematics course in boundary value problems has survived and still stands as a basic service course in the undergraduate curriculum.

Because PDEs are central to mathematics, both pure and applied, their universality in applications, particularly new ones, will continue to influence the curriculum, and therefore we should expect to make more shelf space in our bookcases. Moreover, courses in advanced PDEs have become standard offerings in many graduate programs across the country. Early texts, some of which are still in print, authored by individuals like Garabedian, Hellwig, or Petrovsky, dealt with theoretical aspects of the subject, like existence and uniqueness of solutions. These evolved into a new generation of texts that bring modern analysis and functional analytic techniques to bear on the problems. In recent years, spurred by exciting new applications and nonlinear phenomena, as well as the advent of the computer, even more diverse texts have appeared. The idea that PDEs are models for all sorts of real phenomena has spawned a near renaissance. Problems in heat transfer, electrodynamics, fluid mechanics, and mechanical vibrations have made room for problems in population dynamics, finance, physiological modeling, and other nontraditional applications outside the engineering and physical sciences. Therefore, at the present time there are not just one or two PDE courses taught in universities. Beyond the standard undergraduate course is a myriad of courses, each of which focuses on different aspects of the subject. To complement these courses is a large number of diverse textbooks.

This review will focus on four of the newer texts, which are quite different from one another.

The New. One of the most recent texts written for the basic undergraduate course is O'Neil (1999). This text has an attractive format and systematically covers all the techniques using Fourier series and integrals. However, it does not communicate any ideas that would cause students to think that PDEs form a modern tool for examining and modeling physical phenomena. From the viewpoint of applications, which is one of the major driving forces in PDEs, this text is nearly sterile. It is difficult to imagine either students or teachers getting excited about this treatment, whose limited focus is on vibrating strings, vibrating membranes, and heat flow. The book's preface does not clearly set out the author's motives in adding a new volume to our book shelf. It is a book that could have been written decades earlier, and the omission of numerics and modern applications may prevent it from being a serious contender in the textbook market.

In complete contrast is the new book by Ockendon, Howison, Lacey, and Movchan (1999). The authors of this text are practicing applied mathematicians, and they bring applications and modern ideas to the forefront. This book does a first-rate job of communicating what applied mathematics and PDEs are really about, and the authors clearly understand what individuals in these areas need to know.

The book by Ockendon et al. is not a suitable textbook for the standard undergraduate course in elementary PDEs, at least not at most universities in the United States. The authors state that the prerequisites are ordinary differential equations, complex variables, and multivariable calculus, but they omit one important ingredient: mathematical maturity. The level of exposition is more appropriate for graduate students who have some physical experience and who have learned from their experience that mathematics must often be read carefully, with pencil and paper in hand. In some places the analysis is quite brief and readers are referred to the exercises or other resources. There are 20 or so exercises of varying degrees of difficulty at the end of each of the chapters.

Ockendon et al. begin the text with two chapters on first-order quasi-linear equations and then move on to classification and hyperbolic, elliptic, and parabolic equations. Next comes an interesting chapter on free boundary problems, a chapter on non-quasi-linear equations, and a final chapter dealing with a potpourri of miscellaneous topics like Green's functions, solitons, gauge invariance, and quasi-linear systems with one real characteristic.

The book is traditional, but it has a modern perspective with regard to the universality of applications of PDEs. It is a traditional methods text in the sense that no serious mathematical analysis or functional analysis is used. The authors regretted that, to keep the length of the text under control, they were not able to include numerical or perturbation methods. The exposition is typical of classical applied mathematics in that theorems are not formally stated or proved.

My only hesitation in recommending this as a text is that it demands a lot from the reader. There are many topics and ideas packed into roughly 400 pages, and students may find some of the exposition rough going. Nevertheless, the book has an attractive, inviting format. The flow of the main ideas is smooth and to the point. It is a book that delivers a lot of intuition and insight into the subject, and I would strongly recommend adding it to the book shelf. But before selecting it as a textbook, instructors should judge whether their students can handle it; potential users should look at the book very carefully and even read some of the sections.

Yet another recent addition to our library is a second edition of Kevorkian (2000), originally published in 1990 by a different publisher. Those familiar with the first edition will not notice many changes. A few topics, for example, background material on the Dirac delta function and integral transforms, have been relegated to appendices. The book's focus is on analytic techniques that yield a formula, either exactly or approximately. The first two chapters on the diffusion equation and Laplace's equation rely heavily on a Green's function approach. Then, hyperbolic equations are introduced via water waves and gas flow. This approach leads in a natural way to the study of nonlinear equations and discontinuous solutions. A study of applications leading to the Hamilton–Jacobi equation then takes the reader into first-order nonlinear equations, and then quasi-linear systems. The text ends with an excellent chapter on perturbation methods, where boundary layers and multiple scales are introduced (an appendix reviews asymptotic expansions). There is a nice flow from the linear ideas in the early part of the text to the nonlinear ideas encountered in the later parts. Readers should be comfortable with complex variables, which are used from the beginning, and they should have a sound background in multivariable calculus, ordinary differential equations, and elementary aspects of PDEs, like eigenfunction expansions. There are many exercises of varying degrees of difficulty.

Two words that describe Kevorkian's book are "traditional" and "applied." The text presents the classical, analytical techniques used by applied mathematicians, scientists, and engineers to solve problems. Proofs and formal statements of theorems are omitted, but the underlying mathematical concepts are carefully explained. For example, Dirac delta functions are introduced in an intuitive way, and there is no mention of distributions and abstract spaces. In some ways the book defines traditional applied mathematics. Gas dynamics, fluid flow, acoustics, electrodynamics, water waves, optics—it is all here. Certainly the audience should not fear discussions of physical concepts. The Kevorkian text is an outstanding treatment of classical

PDEs and applications suitable for beginning graduate students in mathematics and applied science. It represents what "everyone should know" about PDE methods, except numerical methods, in order to call themselves an applied mathematician. If I had to recommend a single book to a research engineer who wanted to learn the basic, analytical tools of PDEs beyond the standard undergraduate course topics, I might select this book.

The books by Kevorkian and Ockendon et al. are similar in that they address the same audience, graduate students in applied mathematics or in the quantitative sciences, and both lie at the core of applied PDEs. Kevorkian's book does not have the modern outlook of Ockendon et al., but its derivations are more detailed and the methods are more carefully explained. A perusal of these two texts leads one instantly to conclude that Kevorkian has substantially more equations, mathematical formulas, and detail. Ockendon et al., which is about two-thirds the length of Kevorkian, leaves a lot to the reader, and frequently the details are hard to fill in; sometimes there is just a hint of how the idea evolves. Another difference in the texts is the order of the topics. Ockenden et al. immediately launch into a discussion of first-order equations, characteristics, shocks, and so on; Kevorkian first treats the standard second-order equations before dealing with first-order equations. Neither text includes numerical methods, but Kevorkian does include perturbation techniques. I would rank Ockendon et al. a little more advanced because of its breadth and its higher level of exposition. Kevorkian has the edge in carefully developing traditional applications. Instructors may have a difficult time deciding between these two excellent books.

None of the three new books mentioned thus far (O'Neil, Ockendon et al., Kevorkian) give what one would call a theoretical treatment, that is, a treatment that focuses on existence, uniqueness, and other theoretical concepts using functional analytic ideas, or even methods of mathematical analysis. In this category of theoretical textbooks one can find the classic by F. John [12] and excellent books by Evans [7], McOwen [15], Renardy and Rogers [17], and Folland [9]. In a recent review of Evans's text, I described and contrasted the various features of these graduate texts [14].

Another recent theoretical book that seems to have had limited exposure and that I did not mention in my review of Evans [7] is a first-year graduate text by Bassanini and Elcrat (1997). This book is careful and precise about the analysis, but it does not focus heavily on functional analytic methods. It has an attractive format and a good feel for a first, graduate-level course in PDEs. The topics are standard. The first chapter includes material on modeling and classification and is followed by three chapters on the wave equation, the heat equation, and Laplace's equation. These first four chapters can form the basis of a one-semester introductory course. A second course could be based on the topics in the next three chapters, on elliptic PDEs of second order, abstract evolution equations, and hyperbolic systems. The last chapter is a long appendix on distributions and Sobolev spaces; students and instructors can refer to this material as needed, or the chapter could serve as a Chapter 0. Some outstanding problem sets are included.

Although Bassanini and Elcrat state that previous experience with PDEs is not required, it is a little naive to think that this text could be read by someone who has not had an elementary course, unless that student had sound training in analysis and some experience in thinking about physical problems. Nonetheless, it is a serious text that contends with John, Evans, Folland, McOwen, and Renardy and Rogers as an appropriate selection for a beginning graduate PDE course emphasizing theory. A decade ago, it was difficult to find such a text other than John's; it is indeed fortunate that so many have come forward to fill the void.

Another recent contribution to the analysis of PDEs is the text by Barbu [2]. Chapter 1 in this new text reviews, without proofs, topics in measure, integration, and functional analysis. This sets the stage for analyzing a few standard, elementary problems in the framework of Sobolev spaces. Chapters 2 and 3 treat elliptic equations, and the final two chapters examine the heat equation and the wave equation. There is a large number of exercises. Some are routine; some are difficult, but with generous hints. This book definitely has the feel of analysis, and it resembles the portions of Evans, McOwen, and Bassanini and Elcrat where the theory is stressed.

To expand the list of theoretical books one might be tempted to include the older texts by Guenther and Lee [10] or Zauderer [21]. Zauderer is a methods text that is comparable to Kevorkian, while Guenther and Lee, an outstanding text, fills a gap between methods texts and the theoretical texts. It is based on solid mathematical analysis, and theorems and proofs are carefully presented, yet it pays some attention to applications and methods as well. The fairly recent, and unusual, text by Rubenstein and Rubenstein [18] is also in this category.

The Future. At the dawn of a new century we might ask what the future portends. It is clear that a small market for theoretical treatments at the graduate level will persist; applied analysts will always seek to understand the analytic underpinnings of the subject. But, will the standard undergraduate course, which seems firmly entrenched in the mathematical sciences and the curriculum, change its focus? Will traditional courses emphasizing analytical techniques give way to courses that focus on numerical techniques or to mathematical models? Beware answering these questions—remember the CEO who predicted that the time would never come when people would want computers in their homes! It is certainly true that the advent of modern computational devices and software packages has affected our lives and our courses. Most texts and most courses now include a lot of material on numerical techniques like finite differences or even finite elements; many acknowledge the utility of computer algebra packages like Maple or Mathematica or of front-end programs or programming languages like MATLAB. In the last few years there have been a few new titles along the lines of "PDEs with Maple." It is dramatic how the computer can be used to study fundamental questions in both science and mathematics; the speed and efficiency with which problems of great depth and significance can be investigated is often astounding. Students, in spite of their mathematical immaturity, realize the importance of these developments, and they find them exciting. I find in my own undergraduate course that students respond with great interest when we use Maple and MATLAB to develop algorithms that solve real problems in science; eigenfunction expansions often miss the mark of excitement.

A particularly nice book that features a computational approach is Tveito and Winther [20]. Although this book pays homage to traditional analytical methods based on transforms and Fourier series, its focus is finite difference methods. The authors balance both analytic and numerical techniques in the same treatment, and there is also a discussion of nonlinear problems and associated issues like blow-up, energy estimates, and numerics. There are many exercises ranging from straightforward to challenging; some projects are also included. This book is accessible to undergraduate readers and would make a good text for the standard course if the instructor wanted to emphasize numerics but lessen the treatment of modeling. Perhaps this text is a harbinger of future books. A highly positive review can be found in *SIAM Review* [22]. I am tempted to use this book the next time I teach elementary PDEs.

There is another factor influencing the content of our PDE courses. Reform calculus is changing the way students think about mathematics and how they do mathematics. A plague or not, some students come out of reform calculus and postcalculus differential equations courses unable to perform the kinds of technical, analytic derivations and manipulations that we might have expected in the past. (It is not clear that they were ever able to do them in the past either!) I have frequently complained to my colleagues that my juniors and seniors do not know what a convergent series is or how to differentiate an integral with variable limits, and I bemoan our reform efforts for pushing that material into upper division courses. Covering that material takes time away from the central discussion of PDEs. But we all have to play the hand we are dealt, and sometimes that means spending time explaining what *sinh* and *cosh* mean or how a Taylor series might be used to solve Bessel's equation (series methods are not taught in some reform differential equations texts). A supportive view is to argue that the standard undergraduate PDE course gives a real opportunity for students to develop and improve analytical skills that were not honed in calculus. In this sense, the PDE course can play an important role in the undergraduate curriculum. A contrary view might be to shout, "Alas! When will the first reform PDE text appear?"

In any case, the dynamics in the curriculum at the entry level are not, at present, well set, and the influence that numerics or new applications will exert is still uncertain. So, what will elementary PDE books look like in 20 years? Will Churchill remain the standard? Will mathematical modeling be the main ingredient? Will there be greater emphasis on nonlinear models, unlike that in most of our current "linear" textbooks? Further, will numerical techniques begin to replace some of the analytic-formula development? Many will agree that the traditional separation-of-variables course does a disservice to students. One can argue that a beginning course should introduce students to Fourier series and eigenfunction expansions, transform methods, numerical methods, and some nontrivial modeling examples; elementary perturbation methods might also be included. Certainly, a text that omits modern applications or numerical techniques is starting to look dated.

Recommendations. Therefore, many issues influence the choice of an undergraduate textbook. The good news is that there are now numerous excellent texts from which to choose, and many cover a variety of analytic, numerical, and applied topics. The selection ultimately depends upon the instructor's vision and philosophy of the course and the maturity and background of the students taking that course.

I will end with some brief comments on other texts. For the standard undergraduate course, the text by Strauss [19] is an outstanding selection. It maintains a balance between both linear and nonlinear models and problems in one and three dimensions, and it offers an excellent selection of exercises and a good applied mathematics perspective. There is enough theory in the text to whet the appetite of young engineering and mathematics students without being overbearing. A brief chapter on numerics discusses both finite differences and finite elements. An instructor seeking an elementary text in an inexpensive format might examine Farlow [8], a recent Dover reprint that is organized into 46 lessons. Many of these cover numerics; one even discusses Monte Carlo solutions. Beginning students will like the easily digestible outline form of each lesson, where the main point is reached quickly and without ado. Three additional notable texts are DuChateau and Zachmann [6], Haberman [11], and Pinsky [16]. Generally, these are complete, lengthy texts that cover standard analytic techniques (series and transforms), special functions, and numerical methods. A substantial part of DuChateau and Zachmann deals with numerics. Pinsky includes a chapter on perturbations and asymptotics, as well as a long appendix on Mathematica. Of these three, Haberman is the most ambitious and has the widest scope. It contains many modern applications, and a good portion of it is devoted to nonlinear equations. Finally, even this reviewer (Logan [13]) has succumbed to authoring a one-semester, elementary text of about 200 pages.

The classic book by Brown and Churchill [4], now in its fifth edition, still has a following. But, like O'Neil's new book, the omission of numerical methods and exciting applications is a distraction. Ironically, Brown and Churchill, considered years ago as an "applied" text for engineers, is now considered too theoretical for some undergraduate courses because of its willingness to prove various convergence theorems for Fourier series and integrals and to actually verify solutions using uniform convergence!

Two books that bring together PDEs and computer algebra packages are Articolo [1] and Betounes [3]. The goal of Articolo's text is to join Maple and PDEs by developing all of the elementary, formal techniques like separation of variables, and then using the computer algebra package for visualization of the solutions. Many animations are included. The book by Betounes is more sophisticated. It was developed as part of a course in scientific computation in an interdisciplinary program for advanced undergraduates and graduate students. There are advanced applications and even a discussion of weak solutions. Both texts come with a CD-ROM, and both should be examined by instructors who want to structure their PDE courses around Maple or another package.

Finally, this review has discussed only a select group of texts, mostly those from which I have taught or that I have consulted for examples and exercises. There are many excellent books not mentioned here. It is amazing, and satisfying, that the PDE market supports so many outstanding books at both the undergraduate and graduate levels, and so many research-level monographs and specialized treatments. The fact that there have been so many new texts in the last few years is a testimony to the popularity and universality of the subject.

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J. David Logan
University of Nebraska-Lincoln

Statistical Modeling by Wavelets. *By Brani Vidakovic.* John Wiley, New York, 1999. $79.95. xiii+382 pp., hardcover. ISBN 0-471-29365-2.

Wavelet analysis has had a profound impact on the applied sciences, including signal analysis, image resolution, and statistical modeling. Indeed, it is expected to be applicable to any data compression problem in which the important notion of scaling plays a role. It is the multiscale nature of wavelet analysis that makes its use so versatile. In contrast to Fourier analysis, multiscale decompositions of functions give a quantitative meaning to the localization of frequency ranges in the functions. For example, the short-distance behavior of a function near a singularity can be analyzed by "zooming in" on the singularity with appropriate translates of wavelets having successively higher frequency scales represented by successively smaller length scales. If one tries to understand the singularity as a distribution over Fourier modes, the appropriate high-frequency bands of that function have poor localization.

In his book *Statistical Modeling by Wavelets*, Brani Vidakovic gives an introduction to both wavelet analysis and the application of wavelet decompositions to statistics. His discussion of the fundamental aspects of wavelets is not intended to be a logical development with a complete collection of mathematical proofs. Instead, he explains the most important concepts and illustrates them with good examples. To be sure, he states lemmas and theorems—even proves some of them—but his purpose is to provide a bird's-eye view of wavelet analysis to the experts in his own field. For each result that he states without proof, he provides a reference for the benefit of the logical reader. Since this reviewer is not an expert on statistical modeling, it is difficult to compare the application of wavelet methods to the application of older methods. For any set of problems demanding phase space localization, however, the wavelet transform has usually been a distinct improvement over the windowed Fourier transform. In any case, the very application of wavelets to statistical problems as described by Vidakovic has definitely changed the face of statistical modeling. Since the use of wavelets has been known to significantly alter the pedagogical approach to other areas, we also expect future introductions to this area to be affected accordingly. The concept of multiscale analysis changes the way in which problems are formulated and understood.

The importance of scaling cannot be emphasized enough. In the field of image resolution, for example, the interpretation of an image can actually be affected by scale. When we interpret some cloud formation as a dragon, a subtle change in our point of view can alter the image to that of a dolphin. That shift in viewpoint is an unconscious shift in our emphasis on details, often with the former set of highlighted details having a length scale different from the length scale of the latter set of details.

Vidakovic gives an excellent review of the various kinds of wavelets that have been developed, including early multiscale constructions such as the Haar basis and the Franklin basis. He describes the Meyer wavelet, the construction of which surprised the mathematical community. Indeed, this wavelet convinced many signal analysts and harmonic analysts that a variety of regularity and decay properties were possible for wavelets, and so there was soon a booming industry in the construction of wavelets that previously were not even believed to exist. The author describes these subsequent developments, including the discovery of Lemarie wavelets, the incorporation of many constructions into a general framework known as multiscale resolution analysis, and the remarkable construction of Daubechies wavelets. He discusses the issue of asymmetry in Daubechies wavelets and covers the Daubechies–Lagarias algorithm as well. More generally, Vidakovic describes the connection between multiscale resolution analysis and cascade algorithms. With regard to the construction of additional kinds of wavelets, he carefully reviews the biorthogonal construction of Cohen, Daubechies, and Feauveau. In the same spirit, he introduces the semiorthogonal wavelets, including the Chui–Wang wavelets. Beyond this, the author explains the theory of wavelet packets and their use.

This book is excellent for any expert in statistics who needs a quick, intensive introduction to wavelet analysis. Moreover, the careful and thorough referencing is a very good work of scholarship.

GUY BATTLE
Texas A & M University

Analytical Theory of Biological Populations. *By Alfred J. Lotka, David P. Smith, and Helene Rossert.* Plenum, New York, 1998. $49.50. xxxi+220 pp., hardcover. ISBN 0-306-45927-2.

Alfred Lotka is the father of mathematical demography. His papers written in the early decades of the twentieth century laid the foundations for the modern theory of demography, and his book *Elements of Physical Biology* (Williams and Wilkins, Baltimore, 1924), later reprinted as *Elements of Mathematical Biology* (Dover, New York, 1956), is a classic. However, no complete collection of his papers has been published. The most comprehensive collection of his work is found in his planned book *Théorie Analytique des Associations Biologiques*, two parts of which were published in *Actualités Scientifiques et Industrielle* (Hermann, Paris). (A third part, based on *Elements*, was never written.) The first part (*Principes*) appeared in 1934 and the second part (*Analyse Démographique avec Application Particulière à l'Espèce Humaine*) appeared in 1939. *Analytical Theory of Biological Populations* is a translation of these two works. It provides, for English readers, Lotka's own presentation of his contributions to mathematical demography and theoretical ecology.

The book has two parts. In the short Part 1, Lotka describes his philosophical and scientific points of view regarding the analysis of the dynamics of biological populations. In these musings, his comments range from the direction of time, free will, and man's special place in the family of biological populations to energetics, uncertainty, and the comparison of biological populations with inorganic populations of chemicals. He emphasizes the importance of multispecies interactions and includes a general description of ecosystems in terms of systems of differential equations. This mathematical approach accounts for the association of his name with that of his contemporary, Vito

Volterra. While his mathematical treatment of these systems is cursory in this book and sometimes peculiar from a modern point of view, it does provide insights into the thinking of this important figure in the history of theoretical ecology.

Part 2 constitutes the bulk of the book. Here the focus is on a single population of individuals classified according to chronological age, with an emphasis on human populations. In the first four chapters Lotka develops the fundamentals of age-structured population dynamics. He defines fundamental terms, such as intrinsic growth rate, stable age distribution, birth and death rates, mean age of a population, moments and cumulants of survival curves, total reproductive value, etc., and he derives relationships among these variables. Truncated power series expansions are the main analytic tools used to obtain approximation formulas. For the most part, populations in stable age distribution are treated, but some consideration is given to populations with logistic growth. Chapter 5 deals with measures of population growth and studies the relationship between the intrinsic population growth rate and the net reproductive number (expected offspring per individual per lifetime). The final chapters deal with family structure, i.e., the composition of the family as related to marriage rates, birth rates, death rates, etc. Also included is a chapter on lines of descent and, in particular, when they come to an end. Lotka uses population data from Europe and the United States throughout to illustrate his methodology and results.

This 60-year-old book contains dated material, of course, and its translation is mainly a historical contribution. However, the book has aged well and, over and above its historical interest, it could still serve as a solid introduction to the subject of mathematical demography.

J. M. CUSHING
University of Arizona

Semimartingales and Their Statistical Inference. *By B. L. S. Prakasa Rao.* Chapman and Hall/CRC Press, Boca Raton, FL, 1999. $79.95. xiv+582 pp., hardcover. ISBN 1-58488-008-2.

This is the latest in a series of monographs by the author on the subject of modeling stochastic processes. This one looks at the applications of semimartingale theory to statistical inference for such processes, focusing on the fundamental fact that in a wide range of applications, the score vector in a correctly specified model of a stochastic process is a martingale. By combining the central limit theorem and other asymptotic results for martingales with this fact, a theory of asymptotic inference based on the likelihood can be derived. The classes of time series models considered here include discrete and continuous time processes, pure diffusion processes, processes with jumps, and counting processes, embracing most of the applications of interest in the natural sciences, economics, and finance.

The first chapter comprises a detailed survey of the theory of semimartingales, and the book will be found a useful reference by many for this reason alone. Subsequent chapters review the asymptotic likelihood theory, including the important quasi-likelihood case, as well as issues of factorizable likelihoods, partially specified likelihood, and asymptotic efficiency. Whereas a number of accounts of this theory for discrete time series are to be found in the econometrics literature, at least, the extensions to continuous processes with jumps and counting processes are much less widely understood, and this contribution is timely and very welcome.

There is a tremendous amount of material in these densely filled pages, and it would take the reader many hours of detailed study to absorb even a fraction of it. Thus it is all the more remarkable that this is apparently a companion volume to *Statistical Inference for Diffusion Type Processes* by the same author (Arnold, London), also published in 1999. The latter book is referenced here, but without a copy to hand it is not possible to determine how the relevant material is divided between the volumes. A number of topics that might have deserved coverage in this book, including numerical methods and a wider range of applications, are apparently dealt with in the other. One would also liked to have seen more on the treatment of misspecified models, where, for example, the martingale property might fail, but more general constraints on time dependence (e.g., mixingales) might apply. Given the title, though, this would probably be a topic too far removed.

The key question a review must address is the potential readership to whom the book can be recommended. It must be said that this one seems to be aimed primarily at a mathematical audience, rather than to applied workers, practitioners, or nonspecialist students. The style is very terse, and there are many points where more space could usefully have been devoted to motivation and verbal exposition. To cite just one example at random, the discussion of quasi-likelihood (p. 240) makes no attempt to motivate the key concept of optimality or to relate this subsequently to the question of asymptotic efficiency. There are several other points where the excessive formalism of the presentation could leave the reader with applications in mind frustrated. To get the best out of these pages, a reader will need to bring a good deal of preexisting knowledge to bear.

From this point of view, the book suffers by comparison with others devoted to related topics at a similar level. Take the very influential *Martingale Limit Theory and Its Application* by P. Hall and C. C. Heyde (Academic Press, 1980), or as a more recent example, say, *Brownian Motion and Stochastic Calculus* by I. Karatsas and S. E. Shreve (Springer-Verlag, 1988). Neither of these texts is any less advanced than the present one, but both have a good deal more to offer students seeking to acquaint themselves with these fields.

There is also some evidence of haste in the preparation of this book for the press, with rather numerous misprints in the mathematics. These include unmatched or missing brackets, the use of $<M>$ for $\langle M\rangle$ in several places, occasional erratic font selections, and (one fears) maybe some less easily detected slips as well.

Never mind. There is such a rich collection of results on offer here that this book is bound to supply answers in many cases where others fail. It should prove a useful addition to the bookshelf of anyone with a serious interest in time series and stochastic process modeling.

JAMES DAVIDSON
Cardiff University

Elliptic Curves: Function Theory, Geometry, Arithmetic. *By Henry McKean and Victor Moll.* Cambridge University Press, Cambridge, UK, 1999. $29.95. xiii+280 pp., softcover. ISBN 0-521-65817-9.

The preface of this book states that the subject of elliptic curves and elliptic functions is one of the jewels of 19th-century mathematics. This feeling is shared by many. One of the main reasons for this is that the field of elliptic functions and elliptic curves draws from many different areas of mathematics and, conversely, contributes to these different areas. One of the founders of the subject, Jacobi, pointed this out to Crelle (quoted in [1]): "You see the theory [of elliptic functions] is a vast subject of research, which in the course of its development embraces almost all algebra, the theory of definite integrals and the science of numbers." Since then, the list has only grown longer. Andrew Wiles's proof of Fermat's last theorem and the even more recent proof of the modularity conjecture of elliptic curves reiterate how significant these connections are.

Until recently, the only encounters between applied mathematicians and elliptic functions came about because elliptic functions arise quite frequently as solutions of some prototypical nonlinear differential equations: the simple pendulum, the Euler and Lagrange tops, etc. The understanding of elliptic functions required to solve these problems did not go beyond recognizing the differential equation and obtaining its solution from one's favorite table of integrals. A useful, and for many purposes sufficient, list of properties could then be found in the same place. Now the story has changed. Two "new" applications come to mind.

The first one is related to the theory of soliton equations. These equations have a large zoo of solutions that are expressed in terms of functions that are elliptic or that generalize elliptic functions [2]. The geometrical aspects of elliptic curves and their generalizations play a big role in the construction of these solutions. The simplest example is due to Korteweg and de Vries [3], who found periodic traveling wave solutions of the equation that now bears their names. A few of these applications are mentioned in the book.

A second application is elliptic curve cryptography [4]. This is a relatively new (1987) method for public key cryptography. It relies on the fact that on any elliptic curve, one can define the addition of two points on the curve, resulting in a new point on the curve. Elliptic curve cryptography works well because of the complexity of calculating the discrete logarithm on elliptic curves. For this application, the algebraic aspects of elliptic curves are fundamental. Although the last chapter provides all the necessary material for elliptic curve cryptography, it is unfortunately not discussed.

Henry McKean and Victor Moll's goal is to give an introductory exposition to the field of elliptic functions and elliptic curves, emphasizing its connections with complex function theory, geometry, and algebra. This is a noble cause: anyone entering the field might be dazed and confused as to where to start. Many good books are out there, but most focus on one aspect of elliptic function theory. Treating all three aspects mentioned above of course requires the authors to be proficient in these areas. This is a nonissue with the present authors. What is an issue is that this approach also requires the reader to be familiar with, or to want to become familiar with, all three areas. The audience for a book like this is thus restricted from the beginning. The authors are aware of this problem, and the whole first chapter is devoted to a review of the necessary algebraic and geometric ideas. This seems to justify the claim that a first acquaintance with complex function theory is all that is required to be able to read this book. My impression, however, is that these first 50 pages will be meaningful only to those who are already familiar with them. The first chapter is in this sense more a warning than an invitation.

A review of this book aimed at an audience of pure mathematicians is bound to be very positive, given its scope and the number of connections it brings out and emphasizes. Writing for an audience of applied mathematicians makes me more cautious. The background of the typical applied mathematician does not contain much abstract algebra, differential geometry, projective geometry, etc.

Most applied mathematicians will find themselves comfortably at home in Chapter 2, which uses complex function theory almost exclusively. This chapter contains a classical description of elliptic functions, mainly following Jacobi's approach. One should not expect to be able to breeze through this or other chapters effortlessly. Throughout the text, the authors have sprinkled many examples and exercises. The examples are illustrations of previously discussed ideas. More often than not, the arguments in the examples are not complete and the examples are followed by one or more exercises asking the reader to complete the arguments. Most exercises in the book are of this nature: they want to bring a point across, which is good. Unfortunately, I think this is done too frequently: a devoted reader who wants to understand the whole picture soon realizes that these exercises should not be skipped. The next realization is that many of them are far from straightforward. At best this makes for slow reading, at worst, for painful reading. On the other hand, any author who is willing to offer the reader 25 cents for trying (p. 86) needs to be cheered (even though the point of the exercise is that this is not the way to do it).

The next chapter discusses the standard theta functions and brings out many of the remarkable identities they lead to. The material in this chapter should still be accessible to a motivated applied mathematician, although more abstract considerations slowly enter the picture. After Chapter 3, these considerations dominate the discussion. It is the material of these later chapters that brings out all the marvelous connections between this area and other areas. I guess that the typical applied reader will not appreciate their implications, because he or she might find that more background reading is required before entering these chapters.

Throughout the text it is obvious how enthusiastic the authors are about this subject. Adjectives such as "pretty," "spectacular," and "beautiful" are everywhere. Sometimes they lead to frustration, after yet another failed attempt at one of the exercises, but more often they are successful in transmitting the sense of wonder that the authors want the reader to experience.

In summary, this is a wonderful book that should reward those who have the background for it with immense joy and insight. It was not written with the applied mathematician in mind, but if you feel confident about its prerequisites (quite a bit more than the authors claim), you have the opportunity to learn a lot about an area that is slowly becoming more important in applied mathematics.

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Bernard Deconinck
University of Washington

Nonsmooth Analysis and Control Theory. *By F. H. Clarke, Yu. S. Ledyaev, R. J. Stern, and P. R. Wolenski.* Springer-Verlag, Berlin, 1998. $49.95. 276 pp., hardcover. ISBN 0-387-98336-8.

Four distinguished researchers have joined forces to write an outstanding graduate text: clear, concise, and full of exciting material. The topics of inquiry—and the moderate length of their book—should attract and inspire many a reader. While ignoring so-called postmodernism, the authors have wisely refrained from even declaring their opus modern. They rather stress, quite soberly, a long lineage, dating back to Dini at least (and, in my opinion, to Peano as well). But all the same, the book is distinctly modern in content and orientation. It outlines the rapid developments during the last few decades, takes inventory, and offers emphasis on constructs of proven value and applicability.

Given the dominance ascribed these days to virtual reality, the authors must be commended for explaining that there is nothing virtual, but very much real, about nonsmoothness. In fact, they masterly invite the reader, in Chapter 0, to preview how generalized derivatives creep up in many and diverse settings. Examples include viability of dynamical systems, stability in mechanics, optimality in constrained programming, and implementation of optimal control. Unlike the fashionable deconstruction that still jams the social sciences, these authors are marvelously constructive and concrete in their approach. They travel back and forth on a bridge connecting geometry with analysis, sets with functions. And they travel light, exploring the scenery in depth. Along the way, in four chapters, they offer a wealth of information on proximal calculus, generalized gradients, special topics, and control theory. Their style is easygoing, to the point, and pleasant to read. The feature that nonsmooth analysis thrives in spaces furnished by Hilbert and Banach is not scary here. In fact, diligent readers who are still content with Euclid will also be well rewarded.

Graduate students deserve—and depend on—being offered texts of exceptional value and synthesis. This is one of that relatively rare sort. From this text students will like to learn and professors will enjoy to teach. It takes a central position where (functional) analysis (classical and modern, linear and nonlinear) meets optimization and control theory. Acquiring solid command of the said issues should promise good students abundant yield.

The authors report having struggled with longer, preliminary drafts of the text. I greatly appreciate that they succeeded in making the final version so short and handy. Tight limits require good judgment, difficult selection, and much discipline. A book so rich in content becomes still more recommendable for containing a rich supply of manifold exercises, ranging in difficulty from easy to hard. They make a convincing case for mastering the material.

In short, I like this book very much. It provides powerful tools, it blends theory with applications, and it should be of great interest to every serious student of the field.

SJUR DIDRIK FLAM
University of Bergen, *Norway*

Generalized Riccati Theory and Robust Control. *By V. Ionescu, C. Oara, and M. Weiss*. John Wiley, New York, 1999. $125.00. xxii+380 pp., hardcover. ISBN 0-471-97147-2.

The algebraic Riccati equation plays the central role in steady state optimal control and filtering of linear dynamic systems. Since the original work of Kalman, at the beginning of the 1960s, more than one thousand journal papers have been published on the algebraic Riccati equation. A need was felt for many years for classification and systematic presentation of the most important results about this extraordinarily important equation. That need was partially satisfied with the first book published on the Riccati equation at the beginning of the 1990s [1]. That was an edited book in which several leading authorities contributed chapters, and it produced a nice compilation of many important results on the algebraic Riccati equation known at that time. Except for the chapter written by Basar, all chapters in [1] dealt with the Riccati equation that has a positive semidefinite coefficient matrix for its quadratic term. Another very comprehensive and excellent book that tells the story of the algebraic Riccati equation between the 1960s and the 1990s (the definite sign quadratic term algebraic Riccati equation) is that by Lancaster and Rodman [2].

At the end of the 1980s and during the 1990s, the robust linear system optimal control theory became the dominant research area within control engineering. The robustness results based on the optimization (minimization) of the H_∞ norm of the system transfer function from the system disturbance to the system output were given in terms of the algebraic Riccati equation whose quadratic term coefficient matrix is indefinite. The central theme of the present book on the algebraic Riccati equation, by Ionescu, Oara, and Weiss, is the algebraic Riccati equation that comes from the optimal robust control theory of linear dynamic systems (the indefinite sign quadratic term algebraic Riccati equation, or simply the indefinite sign algebraic Riccati equation).

The book contains three major parts. In the introductory part, the authors review advanced topics in linear algebra and operator theory needed for the study of linear dynamic systems. The presentation in the corresponding Chapters 1 and 2 is at a higher mathematical level than that of many standard textbooks on these topics, for example, [3]. The second part of the book, which presents the algebraic Riccati equation (Chapters 3–6), is "the heart of the monograph." The third part considers the corresponding applications to system theory and robust control (Chapters 7–11). The second and third parts of the book are based mostly on the research papers of Professor Ionescu and his former doctoral students, presently Professors, Oara and Weiss.

The main concern of the book is to provide a comprehensive treatment of the algebraic Riccati equation for the indefinite sign and singular cases within the framework of optimal robust control of linear dynamic systems. The authors have completely achieved that goal and filled a gap that existed in the theory of the algebraic Riccati equation during the last 10 years. Presentation is very general, with no assumptions imposed on the coefficient matrices of the algebraic Riccati equation. The main result of Part II states that "the algebraic Riccati equation has a stabilizing solution if and only if the Toeplitz of the input-output operator of the underlined Hamiltonian system is invertible." The authors treat in parallel both continuous- and discrete-time systems. The presentation is done in a generalized framework of Popov's positivity theory of the 1960s, termed by the authors as the Popov function approach (the Popov function is defined as the transfer function of the Hamiltonian system).

Part III first presents generalizations and extensions of some classic results such as the bounded real lemma, small gain theorem, coprime factorization, quadratic index with constrained dynamics, spectral factorization, and the Nehari problem. In the follow-up chapters of the book, the authors present a new method for H_2-norm optimal control, and study the H_∞ norm control problem without imposing simplified assumptions. In addition, the robust stabilization problem is considered as a suboptimal H_∞ control problem via the use of a singular perturbation technique.

In conclusion, the book represents a very valuable contribution to the algebraic Riccati equation and, in general, to optimal control and filtering theory of linear dynamic systems. It is necessary reading for all researchers and graduate students in engineering and mathematics interested in linear optimal robust control theory.

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Zoran Gajic
Rutgers University

Numerical Linear Algebra for High-Performance Computers. *By Jack J. Dongarra, Iain S. Duff, Danny C. Sorensen, and Henk A. van der Vorst.* SIAM, Philadelphia, 1998. $37.00. xviii+342 pp., softcover. Software, Environment, and Tools. Vol. 7. ISBN 0-89871-428-1.

An earlier book by the same authors [1] explored the relationship between high-performance computing and the numerical solution of dense and sparse linear systems. It was unique in the way it combined treatment of machine architectures with algorithms. The present revised and extended volume brings the computing aspects up to date, contains a much more detailed treatment of preconditioning for linear systems, and extends the numerical treatment to the algebraic eigenvalue problem.

Since [1] appeared many books have been published in numerical linear algebra, often with coverage of high-performance computing or software. However, in my view, the need for the present book has increased rather than diminished. With the continuing developments in theory, methods, software, and architectures, broad overviews are needed, suitable for students and practitioners as well as experts in the area. This book fills this role admirably, covering

- high-performance computing architectures,
- performance analysis and modeling,
- block algorithms,
- solution of dense linear systems, and solution of sparse linear systems by both direct and iterative methods,
- iterative solution of standard and generalized eigenproblems,
- preconditioning.

The book is up to date and has an extensive bibliography. It will be of use to students, researchers, and practitioners.

I finish with an insightful quote concerning the choice of iterative method for solving linear systems (p. 194):

> Because the problems to be solved live in high dimensional spaces, there is no hope that one single method will detect for all sorts of systems where the solution is by inspecting only low-dimensional subspaces with modest computational effort (compared with direct solution methods).

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Nicholas J. Higham
University of Manchester

Geostatistics: Modeling Spatial Uncertainty. *By Jean-Paul Chilès and Pierre Delfiner.* Wiley, New York, 1999. $125.00. xi+695 pp., hardcover. ISBN 0-471-083-151.

The area of spatial modeling and analysis is a very active one, especially among geophysicists, hydrologists, oceanographers, meteorologists, and statisticians, all of whom use methods of spatial prediction and interpolation. They will find the book very useful, containing a wealth of information on a wide range of topics, including in particular kriging in all its variations, linear and nonlinear. The book also contains a discussion of useful families of covariance functions and conditional and unconditional simulation methods for random fields.

Kriging methods are central to the book. Under kriging we find discussions pertaining to simple and universal kriging and cokriging, kriging under inequality constraints, disjunctive kriging, and conditioning by kriging. The emphasis on kriging gives the impression that kriging and its variants are the most useful methods for spatial prediction and interpolation. This, however, is not exactly the case. There are other useful and competitive methods, including Bayesian and smoothing methods—for example, smoothing as described in Rauth and Strohmer (1998), who make use of band-limited functions applied to irregularly sampled points in space. To make my point regarding alternatives to kriging, it is instructive to consider a Bayesian method discussed briefly in the book.

On p. 176 there is a discussion of a Bayesian method of spatial prediction where a predictive density is used. That is, suppose it is desired to "predict" the value of $Z(\mathbf{s}_0)$ at location $\mathbf{s}_0$ from data $\mathbf{Z} = (Z(\mathbf{s}_1), \ldots, Z(\mathbf{s}_n))$ at locations $\mathbf{s}_1, \ldots, \mathbf{s}_n$. The Bayesian predictive density approach provides, under assumptions, the conditional probability density $f(z(\mathbf{s}_0)|\mathbf{z})$ of $Z(\mathbf{s}_0)$ given $\mathbf{Z} = \mathbf{z}$. The method has been used successfully by Handcock and Wallis (1994), and its extension involving parametric transformations similar to trans-Gaussian kriging has been studied in De Oliveira (1997) and compared with ordinary and trans-Gaussian kriging in Kozintseva (1999). The last study shows that ordinary kriging, trans-Gaussian kriging, and the (modified) Bayesian method all give very similar predictions as judged by cross validation and mean squared error, but that the difference lies in the prediction intervals. Generally speaking, the Bayesian method gives wider but more realistic prediction intervals that capture the true $Z(\mathbf{s}_0)$ with a higher probability. In this connection, the predictor of the Bayesian method is the median of $f(z(\mathbf{s}_0)|\mathbf{z})$ because the mean $E(Z(\mathbf{s}_0)|\mathbf{Z})$ does not exist for certain priors, a point overlooked by the authors.

In general, kriging may have a problem providing realistic prediction intervals when the data are skewed, and geophysical data are quite skewed more often than not. The remedy of $\pm 3\sigma_k$ on p. 177 is puzzling.

Another puzzling fact is that in a book whose whole purpose is to deal with spatial random fields there are time series plots, two-dimensional images, all kinds of other plots, and even cartoons (about 160 figures in total), but not a single three-dimensional (3D) plot of a random field realization. Such 3D plots are needed to better motivate possible applications of the parametric correlation functions discussed in Chapter 2. For example, it would have been illuminating to see various 3D realizations corresponding to the Matèrn correlation (K-Bessel model) on p. 86, for different parameters. Who can understand the meaning of the dry formula of the K-Bessel correlation for various parameters? Is the model appropriate for smooth realizations? Or probably very oscillatory? Perhaps even for both? It would have been helpful to include 3D figures such as Figure 1. Similar figures can be generated online at http://www.math.umd.edu/~bnk/bak/generate.cgi using a program written by Boris Kozintsev. The algorithm follows a circulant embedding method discussed in Chan and Wood (1997) and Kozintsev (1999).

In general the style is clear, although in places throughout the book the authors resort to providing the reader snippets of information on various topics in fine print. This is probably unavoidable given the very large number of topics covered in the book. One may get frustrated at times over the inability to distinguish clearly between scalars

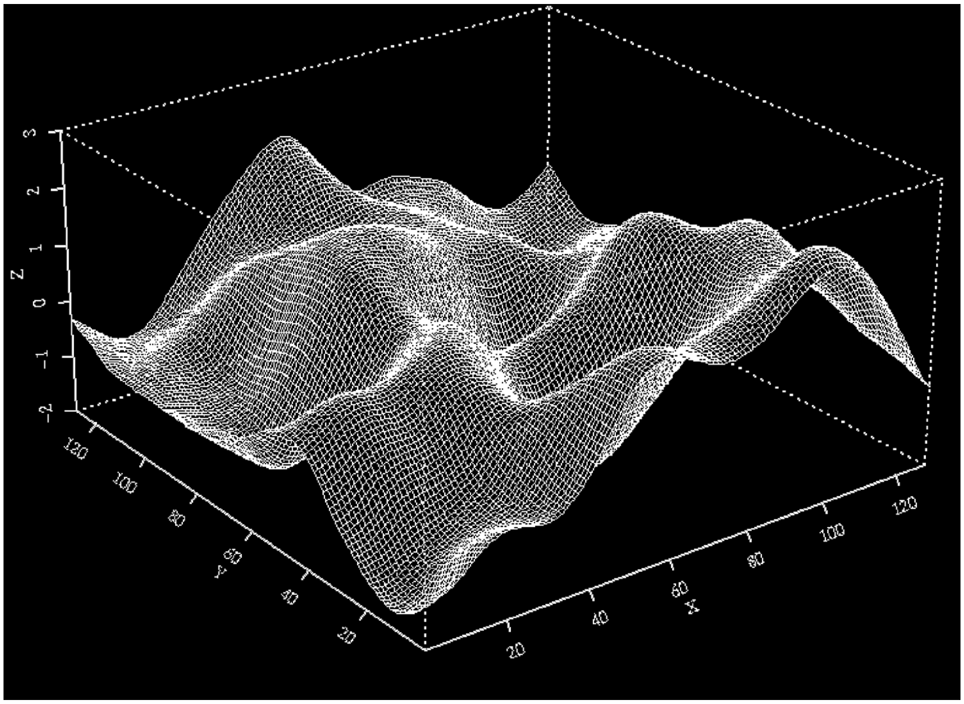

Fig. 1 *A* 128×128 *realization from an isotropic Gaussian random field with Matérn correlation with parameters* $a = 7$, $\nu = 4$.

and vectors. There are places, however, where this is corrected by the use of boldface letters to denote vectors and matrices.

Much of the book is indebted to the work of Georges Matheron, founder (in 1967) and head (until 1996) of the Center of Geostatistics at the Paris School of Mines, and to the experience of the authors gained at that center. Despite of the indicated shortcomings, the book is a very useful addition to the existing literature on spatial processes.

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Benjamin Kedem
University of Maryland

Neural Networks and Analog Computation—Beyond the Turing Limit. *By Hava T. Siegelmann*. Birkhäuser, Basel, 1999. $49.50. xiv+181 pp., hardcover. ISBN 0-8176-3949-7.

There has been a growing interest in the study of neural networks and the development of algorithms based on neural networks. There have been a number of applications in pattern recognition, engineering optimization, operations research, and gene sequencing using neural network algorithms. On the other hand, issues related to computational complexities of neural networks are not normally afforded much importance. The aim of this book is to fill this void. Siegelmann's book focuses on the computational complexities of neural networks and making this research accessible. To a great extent this book accomplishes the said task nicely.

In the first two chapters, the author sets up the analog model for neural networks. A brief introduction to traditional automata, computability, and complexity theory is provided. This chapter is beneficial to one who is unfamiliar with traditional complexity issues.

In Chapter 3, networks with rational weights are discussed. This chapter shows the equivalence of Turing machine computation with networks of rational weights. The equivalence proof is done through simulation.

Chapter 4 describes networks with real number weights. The real number weights are appealing for modeling analog computation. The main result is that neural networks compute functions that are not computable by Turing machines. However, in polynomial time, neural networks recognize exactly the languages belonging to the nonuniform computational complexity class p/poly. But, in exponential time, one can specify a neural network for each binary language. This chapter also gives results on bounded precision.

Chapter 5 describes the hierarchy of computational classes between analog and digital models of computation. This hierarchy is detailed using a variant of the Kolmogorov complexity measure of information. (This is a measure of the quantity of information contained in an individual object.)

In Chapter 6, the weights of the networks are constrained to finite precision. This chapter shows that the resulting networks correspond to Turing space classes.

Chapters 7 and 8 examine the different classes of input functions to neural networks. Chapter 9 discusses networks that exhibit random and stochastic behavior. The randomness arises where each basic component has a fixed probability of failure (this probability can be either a constant or a function of the history of neighboring elements).

Chapter 10 studies the complexities when the neural elements are more powerful. Chapter 11 summarizes analog computation from the neural network model. The penultimate chapter discusses philosophical consequences of computing power under beyond-Turing models.

There are no exercises at the end of each chapter for students to test their understanding. This research monograph will be an ideal companion for a graduate course on the theoretical study of neural networks.

M. KRISHNAMOORTHY
Rensselaer Polytechnic Institute

Reflecting Stochastic Differential Equations with Jumps and Applications. *By Situ Rong.* Chapman Hall/CRC Press, Boca Raton, FL, $69.95. 224 pp., softcover. ISBN 1-58488-125-9.

The rising interest in modeling uncertainty in dynamical systems has emphasized the need for understanding the behavior of stochastic differential equations (SDEs). In an attempt to model noise of different types in a variety of applications, an increasing number of models are given by SDEs with jumps. The author takes models of population dynamics and neurophysiology, in which the solution cannot take on negative values, as the motivation for studying reflected SDEs with Poisson jump processes. The book is mainly concerned with providing existence, uniqueness, convergence, and comparison results for reflected SDEs with jumps. While these results are analogous to those previously given for other processes, such as nonreflecting processes and processes without jumps, the case of jumps with reflections requires special consideration. Variations in models are covered by considering a variety of cases for the coefficient in these SDEs.

Chapters 1 and 2 give background material on nonreflecting processes, which is used in later chapters. Most of it references other works or results of the author from the 1980s. In Chapter 1 results for processes composed of martingales and Poisson jumps without reflection are reviewed. These are mainly concerned with local time (occupation time near a certain value) and a generalization of Ito's formula. Theorems for existence, uniqueness, convergence, and comparison of solutions are given. In Chapter 2 results for Skorohod problems with jumps are given, which are then used in the following sections as necessary.

In Chapters 3–6 the results for reflected SDEs with jumps are given. Chapter 3 deals mainly with existence of weak and strong solutions for various types of coefficients: continuous, discontinuous, random, nonrandom. Here continuous reflection on general convex domains and jump reflection on the half-space are considered. Chapter 4 is concerned with comparison, convergence, stability, and uniqueness, and some results are limited to certain types of domains. Chapter 5 discusses nonlinear filtering for reflecting SDEs with jumps, and a Zakai equation is derived for the conditional probability density in a specific case. Chapter 6 discusses necessary and sufficient conditions in the context of stochastic control for reflected SDEs with jumps. Chapter 7 gives a brief demonstration of how the results can be used in classes of equations that appear in applications.

This book provides a large collection of theoretical results for reflecting SDEs with jumps. Most of the material in Chapters 3–7 is the work of the author, and many results are new. A large list of the author's work is given in the references. This research monograph will be useful to those looking for a uniform presentation of the theoretical results for reflection SDEs with Poisson jumps. It is mainly directed to probabilists in the field. Following a short preface giving the motivation for studying reflections, the material launches into a standard theorem-proof format. This is the dominant format of the book, with short remarks following the proofs, usually indicating results if certain conditions are altered. Definitions and intuition are not given for standard notions in stochastic processes, so readers wishing to learn about the field will not find the book informative. Application and interpretation of the results in the context of mathematical models is mainly limited to a short section at the end of the book.

The format of the text is quite dense, since it appears to be typeset in TEX, rather than LATEX. The equations are not displayed, there are no figures, and, in general, there are only brief introductory and discussion paragraphs. Even for this series in *Research Notes in Mathematics*, the style is very compact. This makes for intense reading, even for specialists. While the typesetting is not particularly friendly, it does allow for the inclusion of details and additional remarks concerning alternative conditions for some of the results. Those readers who want to dig into the proofs of the results will find plenty of material. There are some misprints in some important theorems, but this should not present a problem for those in the field.

The applied audience may find this book disappointing, since the "and applications" part of the title is at best misleading. There is little explicit connection with applications, except for a short section at the end, where the theoretical results are applied to systems of a certain class. The references for these equations are a few papers from the 1980s on population control and neurophysiological control. There is one paragraph on how the existence, stability, and comparison results can be interpreted in terms of a population model. No modeling issues are discussed. Another omission of this book is a relation of the results to other recent work on reflecting SDEs or SDEs with jumps. There are few references to papers in the 1990s, outside of the author's work and standard texts on stochastic processes. Although reflecting stochastic processes and jump processes have been used in modeling, no recent examples of models are discussed. However, if one is looking for theoretical results for a particular model of this type, this book would be an obvious place to look, given its level of detail.

RACHEL KUSKE
University of Minnesota

Theories of Computability. *By Nicholas Pippenger.* Cambridge University Press, Cambridge, UK, and Melbourne, Australia, 1997. $49.95. ix+251 pp., hardcover. ISBN 0-521-55380-6.

Computability is the direct concern only of the last chapter of this book. The other three chapters are shorter and deal with special, but fundamentally important, related areas. Computability is declared to be the study of "whether or not it is *possible* to perform some computation by some particular means (rather than how hard or easy it might be to do so)." The author omits concrete complexity (NP-completeness, etc.), whose results are essentially "quantitative" in character. However, he does include "abstract complexity theory, whose main conclusions are largely qualitative in character."

Accordingly, we have here an excellent exposition of the areas of theoretical computer science that were in vogue before 1970, including many important discoveries made in those areas since 1970. Written from a well-conceived plan, the book constitutes a cogent demonstration that these areas are quite alive. The level of exposition is high, with unformly clear and rigorous proofs. Because the concepts are quite abstract, the book is probably not suitable for most graduate students in computer science. But for exceptional students (those with mathematical proficiency and a taste for abstract algebra) it will make an exceptionally fine text.

The first chapter, "Finite Functions and Relations," features the concepts of lattice and closure, presented abstractly and algebraically. Classes of Boolean functions are considered and the problem area of Emil Post's 1941 book (*The Two-Valued Iterative Systems of Mathematical Logic*, Princeton University Press) is treated in detail, including some recent results.

The second chapter, "Finite Automata and Their Languages," begins with the well-known fundamentals of finite automata theory. It goes on to prove that languages recognized by finite automata are precisely the languages described by a closed logical expression in a certain second-order logical system. It introduces the syntactic monoids of regular languages, in terms of which the subclass of the aperiodic regular languages is characterized. (A regular language L is *aperiodic* if, for some positive integer k and for all words x, y, and z, the word xy^kz is in L if and only if $xy^{k+1}z$ is in L.) Aperiodic languages are also characterized in terms of regular expressions and formulas of symbolic logic. Also covered in Chapter 2 are: the concept of a *variety* of languages corresponding to the concept of a variety of monoid; recognizability of classes of integers by finite automata; and the use of finite automata, with and without pebbles, to run mazes and to analyze two-dimensional arrays.

The third chapter, "Grammars and Their Languages," begins by covering the well-known concepts: context-free grammars, general grammars (which generate all recursively enumerable languages), linear grammars, ambiguity of context-free languages, etc. But then a certain language is constructed that is not recursively enumerable; the proof by the diagonal method occupies several pages, setting the context for a general discussion of undecidability results. A rigorous treatment of the topic of families of languages includes the notion of a *rational cone* of languages, i.e., a family closed under (1) homomorphic reflections (otherwise known as inverse homomorphisms), (2) intersection with regular languages, and (3) homomorphic images. For example, the families of context-free languages, recursively enumerable languages, and linear languages are all rational cones. In this context several other concepts are discussed relating to language homomorphisms and other similar operations. The final section discusses generating functions for context-free languages, regular languages, and linear languages.

The fourth chapter, "Computable Functions and Relations," presents the theory of computability proper, including recursive functions, recursive sets, and recursively enumerable sets. It begins with an abstract definition of what are called "reflexive classes" (of partial functions over the nonnegative integers). This abstract definition embodies ideas that, in most books on computability theory, appear only after much exposition. A teacher using this book as a text would do well to revise the order of presentation at this point, introducing the concept of reflexive class only after the student has had a chance to grasp the component ideas separately and concretely. Ultimately, the abstract concept will afford the accomplished student many fruitful insights.

After some intensive development in Chapter 4, it is proved that what is generally known as the class of partially recursive functions is a reflexive class and indeed the smallest such class. Thereupon follows the customary development of computability theory: the unsolvability of the halting problem, Rice's theorem, and the fixed-point theorem (otherwise known as the recursion theorem); Ackermann's function; indexing of recursively enumerable sets; etc. The lambda calculus and Scott–Strachey semantics are also woven in. The notion of a recursive cone of functions, vaguely similar to the notion of rational cone of Chapter 3, is introduced; it is proved that the smallest recursive cone is identical to the smallest reflexive class (i.e., the class of partial recursive functions).

Chapter 4 gives a good account of much of the research that has been inspired by Emil Post's 1944 paper [1]. For example, there is a three-page proof of the Friedberg–Muchnik result that there exist two recursively enumerable sets, neither of which is Turing-reducible to the other. The chapter ends with a consideration of abstract complexity theory and of computable real numbers (the topic that motivated Turing's famous 1936 paper).

The book, which has a list of about 200 references, is well indexed, well organized, and quite well written. But the strongest commendation is for its selection of material and its coherent organization.

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Robert McNaughton
Rensselaer Polytechnic Institute

Iterative Methods for Optimization. *By C. T. Kelley*. SIAM, Philadelphia, 1999. $37.00. xv+180 pp., softcover. Frontiers in Applied Mathematics. Vol. 18. ISBN 0-89871-433-8

The author of this well-crafted work on optimization algorithms has selected his material with considerable care and presented it in an elegant and tasteful manner.

The book addresses a narrow but fundamental problem: the minimization of a finite-dimensional smooth, nonlinear function that either is unconstrained or has simple upper and lower bounds on its variables. Emphasis is placed on the theoretical formulation and detailed analysis of a selected subset of algorithms, along with their numerical illustration. A unique feature, which distinguishes this book from others that cover the same problem area, is an extensive, unified discussion of algorithms for noisy problems, i.e., problems defined by perturbations of smooth functions. Almost a third of the book's length is devoted to this important topic.

The encompassing literature on optimization and equation-solving is vast and has undergone a profound restructuring during the past decade, following the so-called Karmarkar algorithmic revolution. Thus it will be useful first to place the work under review within this broader context. Next, we discuss its underlying philosophy and give an overview of the book's contents. Some distinguishing features are then itemized. Finally, in concluding remarks, we make recommendations for usage.

Context. Prior to the publication of the landmark paper by Karmarkar [2] in 1984, the field of algorithmic optimization was marked by a prominent watershed between linear programming (LP) and nonlinear programming (NLP). The polyhedral structure of the feasible region of a linear program, and the fact that its preeminent solution engine—Dantzig's simplex algorithm—proceeded along a path of vertices, imparted the flavor of combinatorial or discrete mathematics to LP. In contrast, NLP, where the solution methods are often rooted in analytic techniques associated with classical mathematicians, for example, Newton, Cauchy, Lagrange, and Euler, retained the flavor of continuous mathematics. Hence the LP/NLP division.

The subject of nonlinear programming was often subdivided again. On one side lay unconstrained optimization—closely allied with nonlinear least squares minimization, solving systems of nonlinear equations, and finding a global minimum of a nonlinear function—and extensions to include linear equality/inequality constraints. On the other side lay problems with more general nonlinear equality/inequality constraints, including the special case where the nonlinear functions used to define the objective and constraints are convex.

We shall call the foregoing groupings of problem areas and associated algorithms the *classical treatment* of the subject.

Following a decade of intense research motivated by Karmarkar's 1984 article, a new portrait has emerged that has left virtually none of the aforementioned problem areas untouched. This is summarized in Figure 1. (Much can be said to justify this dramatic reorganization of the field of differentiable programming, but this is not the appropriate place for an extended discussion.) Within the *post-Karmarkar treatment* depicted in the figure, observe that unconstrained minimization (UM) and nonlinear equation-solving (NEQ) are placed on *opposite* sides of the convexity-nonconvexity (of constraints) watershed. Together they form the foundation and twin pillars of the field. The two facets of the subject are bridged by univariate optimization and equation-solving, whose algorithms are much more closely interwoven than their multidimensional counterparts, and by linear programming.

Kelley has shown prescience in writing two *distinct* but interrelated books—the one under review and an earlier companion work on equation-solving [3]—that jointly address the algorithmic foundations of this subject. Both books are very selective in their choice of material, and consequently many important topics are omitted, for instance, an in-depth treatment of nonlinear conjugate gradient–related algorithms for optimization or homotopy-based methods for NEQ. But such topics can be dovetailed in independently, as the need arises. In other words, the overall thrust of the work conforms well with the modern, post-Karmarkar view of the subject as summarized in Figure 1, and the foundation that the two books provide can be extended and built on by the reader in a very natural way.

Philosophy and Overview. Recently, some optimization texts have appeared that are of the "cookbook" variety—a litany of chapters, each treating a class of algorithms in a practical, workaday manner. In contrast, books that are governed by a unifying vision of the subject and underlying principles belong to an alternative, *deeper* variety to which we playfully attach the name "churchbook"—see, for example, Bertsekas [1]. Kelley's book is of this latter type. Throughout, the subject is presented in a unified way, addressing essential ideas, which we now overview.

After an introduction to some basic concepts in Chapter 1, the first part of the book is organized under two key themes: local convergence (Chapter 2) and global convergence (Chapter 3).

In Chapter 2, Newton's method for the UM problem is presented and the intersection with the NEQ problem and Newton's method in that setting is clearly delineated. Several key variants are considered, including finite-difference Newton, Gauss–Newton for both overdetermined and underdetermined nonlinear least-squares (NLSQ) problems, and inexact (or truncated) Newton, where the linear system that defines a search direction is solved by an iterative method, for example, the linear-CG algorithm, with a loose termination criterion. In each case, the local convergence theory is developed in a clear and rigorous manner.

In Chapter 3, under the banner of global convergence, Cauchy's method of steepest descent using a line search is discussed. The nonlinear CG method is developed as an enhancement of steepest descent, but it is only considered briefly. Then the alterna-

Nonlinear Convex-Constrained NCCP: $\min_{\mathbf{x}\in R^n} f(\mathbf{x})$ s.t. $\mathbf{a}_i^T\mathbf{x} = b_i, i = 1,\ldots,m$ $g_j^c(\mathbf{x}) \leq 0, j = 1,\ldots,\bar{n}$	Linear Programming LP: $\min_{\mathbf{x}\in R^n} \mathbf{c}^T\mathbf{x}$ s.t. $\mathbf{a}_i^T\mathbf{x} = b_i, i = 1,\ldots,m$ $l_j \leq x_j \leq u_j, j = 1,\ldots,n$	Nonlinear Programming NLP: $\min_{\mathbf{x}\in R^n} f(\mathbf{x})$ s.t. $h_i(\mathbf{x}) = 0, i = 1,\ldots,m$ $g_j(\mathbf{x}) \leq 0, j = 1,\ldots,\bar{n}$
Linear Convex-Constrained LCCP: $\min_{\mathbf{x}\in R^n} f(\mathbf{x})$ s.t. $\mathbf{a}_i^T\mathbf{x} = b_i, i = 1,\ldots,m$ $l_j \leq x_j \leq u_j, j = 1,\ldots,n$	Univariate Programming (Optimization & Equation-Solving)	Nonlinear Equality-Constrained NECP: $\min_{\mathbf{x}\in R^n} f(\mathbf{x})$ s.t. $h_i(\mathbf{x}) = 0, i = 1,\ldots,m$
Unconstrained Optimization UM: $\min_{\mathbf{x}\in R^n} f(\mathbf{x})$		Nonlinear Equations NEQ: $\text{solve}_{\mathbf{x}\in R^n}\ h_i(\mathbf{x}) = 0, i = 1,\ldots,n$

"The great watershed in optimization isn't between linearity and nonlinearity but convexity and nonconvexity" (Rockafellar [4]).

Notation: m, n, and $\bar{n}$ are integers with $m \leq n$; $\mathbf{x}$ is an n-vector with real components x_j; $\mathbf{c}$ and $\mathbf{a}_i$ are n-vectors; b_i, l_j, and u_j are real numbers (bounds can be infinite); $g_j^c(\mathbf{x})$ is a smooth convex function; and $f(\mathbf{x})$, $h_i(\mathbf{x})$, and $g_j(\mathbf{x})$ are smooth nonlinear functions.

Fig. 1 *Differentiable programming, post-Karmarker.*

tive approach to globalizing an algorithm, based on a trust region, is discussed in detail, again following a unifying paradigm that embraces key variants: unidirectional, Levenberg–Marquardt, and dogleg.

Chapter 4 is the only chapter of Part I that is named for an optimization algorithm, namely, the BFGS quasi-Newton algorithm. Local convergence is discussed in detail and global convergence results are cited in the literature. A concise and to-the-point discussion of limited-memory variants is nicely done. Other quasi-Newton updates are touched on only very briefly in a survey section.

The concluding Chapter 5 of Part I covers optimality conditions for nonlinear objectives subject to simple bound constraints. Extensions of earlier algorithms are described that are based, in particular, on unscaled or scaled Goldstein–Levitin–Polyak–Bertsekas gradient projection.

Part II, which comprises the final third of the book, discusses the solution of noisy problems, namely, objective functions that are perturbations—random or based on the output of an experiment—of an underlying smooth function. The unifying theme, which is introduced in Chapter 6, is that of deterministic sampling, coupled with large-step, forward or central difference gradient estimates over the vertices of a nonsingular simplex, i.e., the convex hull of a set of $n + 1$ points in general position. In Chapter 7, simplex-gradient estimates are used *explicitly* within adaptations of techniques discussed in Part I. Such techniques go under the name of implicit filtering. In Chapter 8, the final chapter of the book, direct search algorithms are described—Nelder–Mead, multidirectional, Hooke–Jeeves—where the simplex gradient arises only *implicitly* within the analysis of convergence.

Particular Features. Let us now itemize some specific features of the book.

- The subject matter is presented in a stylish manner with a great economy of introduced symbols and definitions.
- In Chapter 1, two fairly realistic problems are described, the first arising from discrete optimal control (with variable n) and the second from parameter identification ($n = 2$). These are used at the end of each subsequent chapter of Part I to illustrate the performance of its constituent algorithms. Other simpler problems are also used in Part II. A suite of Matlab codes is available from a web site for purposes of experimentation.
- Algorithms for unconstrained minimization have beautiful properties when applied to a strictly convex quadratic function: conjugacy of search directions, finite termination, the hereditary property of a quasi-Newton relation, and so on. Regrettably, this important intersection between optimization and computational linear algebra is not addressed in the book, but this is eased by its treatment, to some extent, in Kelley [3].
- The discussion of global convergence using a line search relies on Armijo-type step-length and termination criteria and, to a lesser extent, on quadratic and cubic polynomial–based techniques and Wolfe-termination criteria. Armijo rules have the advantage of simplicity and generality. However, they are at a disadvantage when used within the BFGS algorithm, as in Chapter 4, because the condition that guarantees positive-definiteness of the BFGS update cannot be assured.
- The exercises at the end of each chapter primarily serve the purpose of filling in material within the chapter and they are inadequate for instructional purposes.

Conclusions. In summary, Kelley's book succeeds admirably in its stated objectives. It will be useful as a tutorial for self-study by graduate students, researchers and practitioners, and it provides an excellent complement (even antidote) to a "cookbook" treatment of the subject. It can be used as a standalone textbook as well, but, for teaching purposes, it is likely to find its best use as a supplement to a more comprehensive work on algorithmic differentiable programming.

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J. Larry Nazareth
Washington State University

Wavelets. *By Jöran Bergh, Frederik Ekstedt, and Martin Lindberg*. Studentlitteratur, Lund, Sweden, 1999. $44.40. vii+210 pp., softcover. ISBN 91-44-00938-0.

Applications of wavelets include compression algorithms, for instance, to reduce the space necessary to store images electronically or to speed up the transmission of teleconferences or other video signals.

A *wavelet system* is a set of mathematical functions with compact supports or fast decay, specific orthogonality, recursivity, spanning space, and degree of differentiability. A *wavelet* is an element of such a set (as a vector is an element of a vector space). For example, Alfred Haar's wavelets are piecewise constant but discontinuous at finitely many points [4]. Ingrid Daubechies' wavelets are all continuous, and they can form systems with any finite degree of differentiability [1], [2]. Each of Haar's and Daubechies' systems spans the space of square integrable functions. Moreover, tensor products of wavelets provide wavelets suitable for signals in higher dimensions, for instance, two-dimensional images or three-dimensional video signals (images and time).

The wavelet analysis of a signal can begin with an expression (or an approximation) of the signal by a linear combination or a series in terms of wavelets, as with any spanning set in linear algebra. Orthogonality means that the computation of one inner product suffices to determine each coefficient of a signal, as with other orthonormal bases. Recursivity means that there are algebraic relations between the higher and lower frequency wavelets and that such relations reduce the amount of computation necessary to determine the coefficients of the next lower frequency wavelets, as with fast Fourier transforms.

Distinguishing wavelet systems from other orthogonal, recursive, and spanning sets, such as trigonometric or complex exponential functions, the compact supports make it possible to perform different wavelet analyses for different parts of a signal. For example, in an image the compression of an area with fine detail can require many wavelets with high frequency but with supports confined to this area, whereas in the same image a small number of wavelets with lower frequency can accommodate other areas with fewer features, again with supports confined to those areas.

Because continuous compactly supported orthogonal wavelets cannot be infinitely differentiable [3, Thm. 5.1, pp. 1402, 1408], it follows that wavelets cannot lie in the algebra of elementary functions. Consequently, existing constructions and descriptions of wavelets involve some sort of convergence of sequences of functions, uniformly or otherwise. Thus, much of the development of wavelets has involved methods from functional analysis. Therefore, much of the technical documentation about wavelets has hitherto addressed an audience with a solid graduate background in mathematics. In contrast, the book reviewed here—Bergh, Ekstedt, and Lindberg's *Wavelets*—addresses the audience of advanced undergraduates in the mathematical sciences.

"Part I: Theory" begins with signal processing (Chap. 2), which requires only a working knowledge of linear algebra with infinite sequences and series and which introduces concepts that appear in mathematics and engineering with different terminologies. Thus, a complex *signal* is (modeled by) a function $x : \mathbb{Z} \to \mathbb{C}$. For example, the *unit impulse* is the signal δ such that $\delta(0) = 1$ and $\delta(k) = 0$ for every $k \neq 0$. All complex signals form the set $\mathbb{C}^{\mathbb{Z}}$. Moreover, an *operator* on signals is a map $H : \mathbb{C}^{\mathbb{Z}} \to \mathbb{C}^{\mathbb{Z}}$; for example, the *delay* operator is defined by $(Dx)(k) = x(k-1)$, so that n compositions give $(D^n x)(k) = x(k-n)$. Under any conditions of convergence, $x = \sum_{n\in\mathbb{Z}} x(n)(D^n\delta)$, in particular, for every signal $x \in \ell^2(\mathbb{C})$. By definition, an operator is *time-invariant* if and only if it commutes with the delay operator: $H \circ D = D \circ H$. Finally, a *filter* is a time-invariant linear operator. For such a filter H and a signal x, if $h := H\delta$, then $Hx = H[\sum_{n\in\mathbb{Z}} x(n)(D^n\delta)] = \sum_{n\in\mathbb{Z}} x(n)(D^n H\delta) = \sum_{n\in\mathbb{Z}} x(n)(D^n h) = h * x$, which is the convolution of h and x, but here the book does not mention the necessity of conditions for H to commute with infinite series. An instructor could help students sort out the issue. For instance, an exercise at the same level as that of other exercises in the book could ask students to prove that the formula $Hx = h * x$ holds for every *continuous* filter $H : \ell^2(\mathbb{C}) \to \ell^2(\mathbb{C})$. However, a question could then arise as to whether there exist time-invariant linear operators that are defined everywhere yet *not* continuous on $\ell^2(\mathbb{C})$, but that would be an exercise at a level higher than that of the book. The chapter then proceeds with very brief reviews of sampling, two-dimensional signal processing, Fourier series, the Z transform, and Fourier transforms.

Chapter 3 begins with a review of convergence, completeness, and bases in Hilbert spaces, illustrated with the example of the Haar basis, which consists of the elements $\varphi^{(2n)} := D^{-2n}(\delta + D\delta)/\sqrt{2}$ and $\varphi^{(2n+1)} := \psi^{(2n)} := D^{-2n}(\delta - D\delta)/\sqrt{2}$ for every $n \in \mathbb{Z}$. Here signal processing resembles differential geometry in that it focuses on objects that remain invariant under changes of notation. For example, if $x \in \ell^2(\mathbb{C})$ with $x = (x_k)$ in terms of the canonical basis $\mathcal{D} := \{D^k\delta \ : \ k \in \mathbb{Z}\}$, then in terms of the Haar basis, x has coordinates (y_n) with $y_{2n} = \langle x, \varphi^{(2n)}\rangle = \sum_{k\in\mathbb{Z}} x_k \varphi^{(2n)}(k) = \sum_{k\in\mathbb{Z}} x_k \varphi^{(0)}(k-2n) = (x * h^*)_{2n}$, where $h := \varphi^{(0)}$ and h^* is its time-reverse. Similarly, $y_{2n+1} = (x * g^*)_{2n}$ with $g := \psi^{(0)}$. The book points out that one can compute the new coordinates (y_n) by filtering x with h^* and g^* and then downsampling (removing all the odd-indexed values from) the result, but one can be left wondering why one would compute the odd-indexed values $(x * h^*)_{2n+1}$ and $(x * g^*)_{2n+1}$ in the first place. However, the book then demonstrates how downsampling, upsampling, and then the Z and Fourier transforms can be used theoretically to design practical filters with specified properties, in particular, Daubechies' filters, though such filters are not yet connected to any wavelets at this point in the book.

Chapter 4 is where the connections among filters, Fourier transforms, and wavelets are explained, beginning with reviews of the space $L^2(\mathbb{R}, \mathbb{C})$ of all complex-valued, square-integrable functions on the real line and of such related concepts as projections, Riesz bases, and multiresolutions. Here the book does point out the necessity of proofs of convergence to ensure the existence of wavelets corresponding to specified filters, and students with a solid first course in real analysis can find and read such proofs elsewhere [1], [2]. Part I then ends with wavelets in higher dimensions (Chap. 5), the lifting scheme and factorizations (Chap. 6), and the continuous wavelet transform (Chap. 7).

"Part II: Applications" begins with examples of wavelet bases (Chap. 8), including Daubechies' wavelets, symmlets, and coiflets, then presents an algorithm to compute and plot such wavelets and ends with not necessarily orthogonal or not necessarily compactly supported wavelets: Meyer, Battle–Lemarié, and biorthogonal spline wavelets. Chapter 9 continues with adaptive bases, wavelet packets, and best basis selection. The last six chapters (10–15) focus on specific applications. Compression (Chap. 10) begins with a wavelet transform, which yields many coefficients with small magnitudes in nearly constant regions of the signal; then quantization rounds the coefficients to a smaller number of significant digits, which compresses the result but loses information; finally, entropy, Huffman, or some other scheme encodes long strings of zeroes into smaller pieces of information, which completes the compression. Noise attenuation can consist of a mere annihilation of wavelet coefficients with small magnitudes. Fast numerical linear algebra (Chap. 11) demands some knowledge of iterative methods and multigrid methods but is not required for the subsequent chapters. The very short Chapters 12–14 touch upon the differentiability of wavelets, the continuous wavelet transforms, and feature extraction. Finally, Chapter 15 handles boundaries for signals of finite lengths, for instance, through mirror reflections, periodic extensions or padding with zeroes.

A short introduction (Chap. 1) did precede Part I, but I got the impression that it might be difficult for a student to understand without having first acquired a working knowledge of the rest of the book.

Still, in my opinion, Bergh, Ekstedt, and Lindberg have correctly recognized the need for a text on wavelets accessible to advanced undergraduate or perhaps beginning graduate students in the mathematical sciences, and also, in my opinion, they have succeeded in producing such a text for classroom use.

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Yves Nievergelt
Eastern Washington University

Mathematical Topics in Fluid Mechanics: Compressible Models, Volume 2. *By P. L. Lions*. Oxford University Press, Oxford, UK, 1998. $75.00. 348 pp., hardcover. ISBN 0-19-851-4883.

This is the second in a series of books [1] by the author presenting mathematical results on the Navier–Stokes equations and related models. It deals with existence, uniqueness, and regularity theory for both steady and nonsteady solutions of the equations for viscous compressible flow, in which the constitutive law for the pressure is taken to be $p = k^{\gamma}$, with $\gamma \leq 1$. The assumptions on the domain Ω occupied by the fluid are quite general and include the cases of exterior domains, domains with noncompact boundaries, and the full space R^N. The boundary conditions are either Dirichlet or spatially periodic. Most of the new results in the book have been announced by the author in *Comptes Rendus Acad. Sci. Paris*, e.g., theorems concerning global-in-time existence for large data and concerning the existence of steady solutions. This book is intended to furnish extended proofs of these main results and to simultaneously solve several other important, interesting problems.

The book is divided into four chapters. The first is devoted to compactness results for a sequence of approximate solutions. The second and third chapters deal with existence theory for steady and nonsteady solutions. The fourth chapter takes up various related problems, including the cases of a thermally conducting fluid and of an inviscid fluid.

The first chapter is set in the context of the Cauchy problem for the compressible Navier–Stokes equations. These equations form a mixed parabolic-hyperbolic system. The global existence theorems proved in this book are based on a passage to a limit from a sequence of approximate solutions. The core of this chapter consists of new a priori estimates for these approximate solutions, which are insightful and shed new light on the subject. The convergence of a subsequence of the approximate solutions follows from these estimates, using results from Tartar's work on oscillations and compactness and from the author's own work on compensated-compactness.

The second chapter deals with the construction of steady solutions in various types of domains with several different boundary conditions. It can be divided into two parts. In the first part, the existence of time-discretized, isentropic flow is proven using bounds for the L^p-norm of the density. Also, isothermal flow is considered in two-dimensional domains. In the second part, the existence of steady isentropic and isothermal flows is studied. To achieve uniqueness, side conditions are customarily imposed on the density, such as the prescription of the total mass. Other conditions on the density are also used. Several existence theorems are given under hypotheses restricting either the L^∞-norm of the body force or the polytropic exponent. This is followed by a deep discussion about the regularity of the solutions. It is shown that if the data are smooth and the density has a positive minimum, and if both the density and the gradient of the velocity are bounded, then the density must be uniformly continuous.

The third chapter deals with the existence of nonsteady flow. The main results are existence theorems that are global in time for weak solutions of the equations for isentropic and isothermal flows, without restrictions on the size of the initial velocity or external force. The results are obtained under an assumption that the polytropic exponent is sufficiently large, depending on the dimension of the domain. Such results can be compared with the 1934 results of J. Leray for incompressible fluids, which were the first global existence theorems for weak solutions. In considering these results it is well to be mindful of recent examples of Weigant that exhibit the formation of singularities in finite time, in otherwise smooth solutions, even in the case $N = 2$. These examples indicate a very involved and delicate interaction between the dynamical and thermal processes in a gas. Two different approximation procedures are presented in this chapter, one based on regularizations of the problem, and the other on time discretization. The chapter concludes with existence theorems for some generalizations of the basic equations.

The fourth chapter deals with existence theory for nonstationary solutions of several other related model problems: isentropic nonhomogeneous models, semistationary models, Stokes-like models, shallow water models, and other thermal-conducting models. The Stokes problem is particularly interesting, as it already contains most of the primary difficulties of the original equations.

To give a fair assessment of this book, perhaps one should keep in mind the introduction, in which the author states that "all the results are new and this book, in many respects, should most adequately be characterized as a research monograph." On the one hand, the book is indeed full of original and excellent ideas. But on the other hand, not all the ideas are original, and the book is hardly readable for any researcher of intermediate level. Some specific criticisms follow.

(1) Frequently, there are statements without physical meaning. In particular, to the author it is absolutely irrelevant that the density should be positive. In Theorems 6.9 and 6.10, existence is proved within a class of mathematical solutions that may have zero density and nonzero velocity.

(2) The comparisons with earlier results and literature seem to be almost empty. Indeed, if one excludes the comparisons with earlier compactness results of the author, there are very few quotations, despite a lengthy bibliography. This is in fact remarked upon in the introduction, where the author states that "some of the references included there are not quoted in the text." In the opinion of the reviewer, "some" would be more accurately replaced with "most." In fact, in some places old results are proved without making any mention of their origins, even though their origins are listed in the bibliography.

(3) The book is hastily written. Frequently mathematical statements are neither justified nor supplemented with appropriate references, even though appropriate references could be found in the works listed in the bibliography at the end of the book. In some of the theorems and lemmas the hypotheses are not clearly written, and must be deduced from introductory material.

(4) There are numerous misprints that render the reading of the book even more difficult.

In conclusion, this book is one of the most original books written on this subject in recent years. But unfortunately, it will be understandable to only a small number of specialists in applied mathematics with a strong background in functional analysis. For these specialists, it may prove useful as a further step in clarifying and advancing the mathematical theory of compressible viscous flow.

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MARIAROSARIA PADULA
Universitá di Ferrara

Orthogonal Rational Functions. *By A. Bultheel, P. González-Vera, E. Hendriksen, and O. Njåstad.* Cambridge University Press, Cambridge, UK, 1999. $59.95. xiv+407 pp., hardcover. ISBN 0-521-65006-2.

Let $d\mu$ be a finite, positive Borel measure with an infinite set as its support on $[0,\ 2\pi]$. We define $L^2_{d\mu}$ to be the space of all functions $f(z)$ on the unit circle $T := \{z \in \mathbf{C} : \ |z| = 1\}$ satisfying $\int_0^{2\pi} |f(e^{i\theta})|^2 d\mu(\theta) < \infty$. Then $L^2_{d\mu}$ is a Hilbert space with inner product

$$\langle f,\ g\rangle := \frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})\overline{g(e^{i\theta})}d\mu(\theta).$$

The Szegő polynomial $\phi_n(z)$ is defined by the following requirements:

$$\begin{cases} \phi_n(z) = \kappa_n z^n + \text{lower degree terms}, \\ \qquad\quad \kappa_n > 0, \\ \langle \phi_n, z^k\rangle = 0 \text{ for } k = 0, 1, \ldots, n-1, \\ \text{and} \\ \langle \phi_n, \phi_n\rangle = 1. \end{cases}$$

With notation $\phi_n^*(z) = z^n\overline{\phi_n(1/\bar{z})}$, we can write $\kappa_n = \phi_n^*(0)$.

Apart from their significance and profound interrelation with the studies in Hankel and Toeplitz operators, continued fractions, the moment problem, the Carathéodory–Fejer interpolation, Schur's algorithm, and function algebras, Szegő polynomials are also widely used in solving electrical engineering problems. One could see [BC] and [DG] and the references therein for detailed discussion and exposition.

Generalization of Szegő polynomials to rational functions with prescribed poles was first introduced and studied in the early 1960s by Djrbashian [D1, D2, D3] from a pure mathematics viewpoint. Recently, this type of generalization has started to regain the attention of researchers from various areas in solving problems in electrical engineering as well as in mathematics.

Let $\{\alpha_n\}_{n=1}^{\infty}$ with $|\alpha_n| < 1$ be an arbitrary sequence of complex numbers and let some of them have finite or even infinite multiplicity (it is not necessary for them to appear successively). Let

$$b_n(z) := \frac{\alpha_n - z}{1 - \bar{\alpha_n} z}\frac{|\alpha_n|}{\alpha_n},$$

where for $\alpha_n = 0$ we put $|\alpha_n|/\alpha_n = -1$. Next we define finite Blaschke products recursively as

$$B_0(z) = 1 \quad \text{and} \quad B_n(z) = B_{n-1}(z)b_n(z).$$

The fundamental polynomials $w_n(z)$ are given by

$$w_0(z) := 1 \quad \text{and} \quad w_n(z) := \prod_{k=1}^{n}(1 - \bar{\alpha_k} z).$$

The space of rational functions of our interest is defined as

$$\mathcal{R}_n := \left\{\frac{p(z)}{w_n(z)} : \ p \in \mathcal{P}_n\right\},\ n = 0, 1, \ldots,$$

where $\mathcal{P}_n$ denotes the set of all polynomials of degree at most n. It is easy to verify that $\{B_k\}_{k=0}^n$ forms a basis of $\mathcal{R}_n$. Another basis that we shall use is given by $A_k(z) := z^k/w_n(z),\ k = 0, \ldots, n$. So we also have $\mathcal{R}_n = \operatorname{span}\{A_k(z),\ k = 0, \ldots, n\}$. Finally, for any $r \in \mathcal{R}_n$, we define $r^*(z) := B_n(z)\overline{r(1/\bar{z})}$. For each n, we now define the rational version of Szegő polynomials, orthonormal rational functions, $\psi_n(z)$:

$$\begin{cases} \psi_n(z) = \frac{\beta_n z^n + \text{lower degree terms}}{w_n(z)}, \\ \qquad\quad \psi_n^*(0) > 0, \\ \langle \psi_n,\ B_k\rangle = 0, \quad k = 0, \ldots, n-1, \\ \text{and} \\ \langle \psi_n,\ \psi_n\rangle = 1. \end{cases}$$

From the orthogonalization condition $\langle \psi_n, \ B_k \rangle = 0$, $k = 0, \ldots, n-1$, we know that ψ_n is obtained by applying the Gram–Schmidt procedure to the basis $B_0(z), \ldots, B_n(z)$, and in this order. One might as well orthogonalize the other basis $A_0(z), \ldots, A_n(z)$ to obtain another orthogonal basis for $\mathcal{R}_n$. We denote by $\{\Psi_k(z)\}_0^n$ the orthogonal rational functions obtained by applying the Gram–Schmidt procedure to $\{A_k(z)\}_0^n$, and they are uniquely determined by the following conditions:

$$\begin{cases} \Psi_n(z) = \frac{c_n z^n + \text{lower degree terms}}{w_n(z)}, \\ \qquad c_n > 0, \\ \langle \Psi_n, \ A_k \rangle = 0, \quad k = 0, \ldots, n-1, \\ \text{and} \\ \langle \Psi_n, \ \Psi_n \rangle = 1. \end{cases}$$

Write the numerator of $\Psi_n(z)$ as $p_n(z)$. Then $\Psi_n(z) = p_n(z)/w_n(z)$ and $p_n(z)$ is the nth orthonormal polynomial with respect to the varying measure $d\mu(\theta)/|w_n(e^{i\theta})|^2$, as defined in [L1, L2, L3].

Chapters 1 and 2 introduce some fundamental spaces and some classical classes. Notice that the reproducing kernels have played a very important role in the theory of orthogonal polynomials. Chapter 3 discusses the reproducing kernels

$$K_n(z, w) = \sum_{k=0}^{n} \psi_k(z)\overline{\psi_k(w)}$$

in $\mathcal{L}_n$. Both Christoffel–Darbous relations and recurrence relations for the kernels are given in the style of the Szegő orthogonal polynomials. Chapter 4 derives the recurrence for the orthogonal rational functions and the functions of the second kind. The interpolatory quadrature is studied in Chapter 5. The interpolation properties for both orthogonal rational functions and the kernels are discussed in Chapter 6. Chapter 8 discusses Favard theorems of orthogonal rational functions. The authors use Chapters 9 and 10 to cover the convergence moment problems. Chapter 11 examines the boundary situation where the α_i are on the unit circle. Chapter 12 is devoted to the applications of the theory.

This is the first book to discuss the rational orthogonal functions in such a great detail. The book is accessible and will help readers understand the basic theory of rational orthogonal functions. It also proceeds almost immediately to the frontiers of research. It is fresh. Most of the material presented is current and almost none of it previously appeared in book form. The book is a summary of the considerable progress that has been made thus far in the research of rational orthogonal functions. The well-written, cohesive presentation should be of great assistance to new and experienced researchers, both mathematicians and physicists, in the area of rational orthogonal functions.

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VICTOR K. PAN
Barry University

The Cambridge Dictionary of Statistics. *By B. S. Everitt.* Cambridge University Press, Cambridge, UK, 1998. $39.95. viii+360 pp., hardcover. ISBN 0-521-59346-8.

This general dictionary of statistics extends the author's earlier dictionary of medical statistics [1]. The current volume has several nice features. Many of the entries refer the reader either to one of nine statistical texts referenced in full detail on p. viii, or to other texts or articles. Standard and specialized statistical software programs are included, along with the addresses of the publishing companies. Everitt covers applied and theoretical statistical terms for both elementary and advanced analysis. In addition, he includes definitions of relevant mathematical concepts and short biographies of 100 prominent, deceased statisticians. The many figures (134) further illustrate some entries; common abbreviations are included with reference to the appropriate terminology. On the whole, the definitions are clearly and concisely written with related terms cross referenced.

Although the dictionary's coverage is extensive, there is an emphasis on statistics for the medical and biological sciences. Not considering the nine frequently referenced texts, approximately 30% of the references are from biological or medical sources, most typically the journal *Statistics in Medicine*. This biomedical emphasis may come at some cost to other areas of statistical research: I failed to find entries for some sampling terminology and for several terms suggested by a colleague working in geostatistics. Another small criticism is that many of the statistical biographies lack proper references to major works; such references would have been more useful than some of the biographical details the author included.

This dictionary would be most useful to someone who frequently comes in contact with a wide variety of statistical terms and desires easy access to simple definitions, clarifications, or references to sources of further information. My search of the library catalogs of two major research universities found few competitors for Everitt's dictionary; most similar volumes either were published some time ago and consequently lack entries for recent techniques, or aim for a less advanced audience. And the extensive, multivolumed *Encyclopedia of Statistical Science* [2] would entail a much more substantial investment than this $40 text.

Everitt's most serious competition comes from the 1990 edition of the 30-year-old *A Dictionary of Statistical Terms* prepared by Marriott [3]. I compared Everitt's and Marriott's treatments of the letters A and P and found surprisingly little overlap. Of Everitt's 248 entries and Marriot's 322, only 99 were shared! Overall, Everitt's dictionary appears not only more current and biological in focus, but also more applied in its orientation. Everitt's entries include "Angler survey" (used to estimate sport fishing effort and catch rate) and "Person-years at risk," while Marriott includes "Absolute moments" and "Probability integral transformation." Ownership of these two books would provide complete coverage of statistical concepts and terminology. Even alone, Everitt's dictionary should be a worthwhile addition to many personal libraries.

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EMILY SILVERMAN
University of Michigan

Rational Extended Thermodynamics. Second Edition. *By Ingo Müller and Tom-*

maso Ruggeri. Springer-Verlag, New York, 1998. $69.95. xv+396 pp., hardcover. ISBN 0-387-98373-2.

In his classic 1949 paper "On the Kinetic Theory of Rarefied Gases" (Comm. Pure Appl. Math. 2), Harold Grad proposed what has come to be called the 13 moment approximation of kinetic theory. The motivation for the exercise was to solve the Boltzmann equation of kinetic theory:

$$(1) \qquad \frac{\partial f}{\partial t} + \xi \cdot \text{ grad } f = \frac{Q(f,f)}{\epsilon},$$

where $f(x,t,\xi)$ denotes the one particle distribution function for the probability of finding a gas particle at point $\mathbf{x} \in \mathbb{R}^3$ at time t with velocity ξ. $Q(f,f)$ is the bilinear collision operator (nonlocal) and ϵ is a scalar parameter called the Knudsen number. Small ϵ means the space-time scales are such that the gas looks fluid, while large ϵ means that gas looks like an assembly of discrete particles, i.e., the kinetic description.

Solving the Boltzmann equation for given initial data $f(x,0,\xi) = f_0(x,\xi)$ is hard. Hence Grad employed the following clever scheme to reduce the complexity of the problem: Multiply the Boltzmann equation by the vector quantity $(1,\xi_i,\xi_i\xi_j,\xi_i\xi_j\xi_j)$ and integrate over $\xi \in \mathbb{R}^3$. The first equation obtained represents conservation of mass; the second, third, and fourth provide conservation of momentum; and the trace of the fifth yields conservation of energy. This is because the integration of the collision term $Q(f,f)$ with $(1,\xi_i,|\xi|^2)$ is zero in these cases and there are no source terms generated. However, the final integration with $\xi_i\xi_j\xi_j$ does produce a nonzero source term. Employing symmetries, it is easy to see that the procedure generates 13 equations in the independent variables $\mathbf{x}, t$.

The system of 13 equations arising from the 13 moments of the distribution of function f are not closed. Grad chose a closure rule by setting f to be a particular function of the first 13 polynomials in $(1,\xi,\xi_i\xi_j,\xi_i\xi_{jj})$. Thus a closed system of 13 evolution equations reminiscent of the balance of laws of continuum mechanics was obtained. The equations are locally hyperbolic and possess an entropy function.

Grad's system was obtained by a special procedure originating with the Boltzmann equation. It was I. Müller who in 1966 and 1967 realized that the formalism of Grad's system could be taken axiomatically without recourse to the Boltzmann equation. Of course the closure issue remained, and Müller realized that closure could be obtained via constitutive relations, as in classical continuum mechanics, which are restricted via the postulate of an entropy inequality. Müller termed his procedure "thermodynamics of irreversible processes" and, more recently, "rational extended thermodynamics."

The book under review is the second edition of Müller and Ruggeri's earlier treatise of rational extended thermodynamics. It develops the historical outline of the theory in great detail as in the first volume but with much attention paid to recent developments that have occurred since the first volume appeared, e.g., the theory of shock structure, subsystems, new applications. It is well written and contains just about all the details anyone could wish to know about the interplay among continuum mechanics, kinetic theory, and systems of relaxation equations.

M. Slemrod
University of Wisconsin-Madison

Compartmental Modeling with Networks. *By Gilbert G. Walter and M. Contreras*. Birkhauser, Boston, 1999. $49.95. xviii+250 pp., hardcover. ISBN 0-8176-4019-3.

This slim book is divided into four parts, beginning with networks and graphs, followed by Markov chains, definitions, and examples of compartmental models, and concluding with some theory of compartmental models. A short list of exercises accompanies each section. An appendix on "Mathematical Prerequisites" reviews basic matrix theory, systems of linear constant-coefficient differential equations, and elementary Maple linear algebra commands. The authors intend the book as a text for an undergraduate-level course on mathematical modeling for students who have had calculus and a semester of matrix theory. Given the theorem-proof format of much of the book, my guess is that this is rather optimistic. The four parts are reviewed separately.

"Part I. Structure of Models: Directed Graphs." For someone whose graph theory background is rather minimal, I found this part of the book to be the most inspiring. The basic definitions of graph and digraph are introduced. Topics such as connectedness properties of graphs and digraphs and the problem of finding a strongly connected orientation for a graph are treated. We learn how to solve the minimum connector problem using the greedy algorithm. For example, in a railroad network joining a number of cities, we would like to keep those portions that connect all the cities but are least expensive to maintain. Tournaments, planar graphs, and adjacency matrices corresponding to directed graphs are treated. Helpful examples are provided that illustrate various mathematical points, but no meaningful applications are treated in any detail in Part I. A very brief concluding section describes simple Maple commands associated with graphs.

"Part II. Digraphs and Probabilities: Markov Chains." Here we are introduced to Markov chains with almost no mention of probability theory (e.g., conditional probabilities) nor use of its notation. Multiplication of the probability vector at time t by the transition matrix, we are told, results in the vector of probabilities at time $t+1$. Separate chapters treat regular and absorbing Markov chains. The powers of the transition matrix converge to a positive stochastic matrix in the regular case so, in the long term, the probabilities of being in the various states are independent of the initial probability vector. Absorbing Markov chains are put into canonical form, from which one can answer two basic questions: What is the probability of entering a given absorbing state? and How often will the chain be in a given (nonabsorbing) state before absorption? Finally a brief example is given concerning cross-breeding strategies in genetics.

"Part III. Compartmental Models: Applications." In this part, the presentation changes dramatically from the theorem-proof format of previous parts to a more informal style. The aim is to introduce examples of compartmental models. These include the logistic equation, tracer kinetics for the flow of a drug through a biological system, and forest ecosystem models. Separate chapters treat simple epidemic models, host-parasite models, the Leslie matrix model of an age-structured population, fisheries models, and drug kinetics. Except for the Leslie model, which is a discrete-time model, the others result in linear or nonlinear ordinary differential equations.

While this is an impressive list of topics, covered in a mere 60 pages, with very few exceptions the presentation consists of writing down model equations with very little motivation and moving on to the next model. The few occasions where some analysis is carried out are due to the existence of closed form solutions to the equations.

"Part IV. Compartmental Models Theory." Linear compartmental systems typically take the form of a linear system of ordinary differential equations $x' = Cx$, where C is a quasi-positive matrix (nonnegative off-diagonal entries) with nonpositive diagonal entries, diagonally dominant in the sense that the column sums are nonpositive. The components of the nonnegative vector x represent the mass, concentration, or density of the "material" in the various compartments. Quasi-positivity of C is shown to imply nonnegativity of the matrix exponential $\exp(Ct) \geq 0$ $(x(t) \geq 0)$, and Gerschgorin's theorem together with diagonal dominance leads to the conclusion that the nonzero eigenvalues have negative real parts. A number of positivity results are proved, among which we note only that the matrix exponential has all positive entries when the associated digraph is strongly connected. Mentioned in passing, without proof, is the fact that the zero eigenvalue of a strongly connected compartmental matrix with zero column sums (hereafter, an SCCM) is simple. Instead, the authors show that the variance of the $n-1$ nonzero eigenvalues of an n-by-n SCCM matrix, C, can be bounded above and below in terms of the traces of C and C^2. The importance of this result is not well motivated, and the result plays almost no role in later developments.

Concluding chapters touch on various special topics. One treats system identifiability for compartmental systems $x' = Ax + Bu$ with inputs and observations $y = Cx$. The goal is to identify A from knowledge of $u(t)$ and $y(t)$. Laplace transform methods are used to illustrate the issues and to obtain a negative result based on the structure of the system. The final chapter introduces various measures of system complexity in an ecological setting. One of these indices is related to the Shannon index of information theory. A brief discussion of the relation of system complexity and system stability indicates that these two attributes increase (decrease) together.

The book has its problems. There are many misprints. The authors appear to have an aversion to referencing earlier results to support their assertions. This makes many proofs difficult to follow.

I found myself disappointed that more current examples of compartmental systems were not included, that many of the models introduced were not well motivated, and that nothing much was done with the model equations. Part III is essentially disjoint from the remainder of the book—the theory of networks is not employed, and the examples treated there are not resurrected in Part IV for further analysis. Compared with the other parts, the mathematical level of the presentation is markedly lower.

Several proofs have minor flaws. For example, in their proof of the classical result that the nth power of a regular Markov matrix converges to a Markov matrix with identical columns, the authors' iteration argument is flawed because an estimate is not independent of the starting vector. Proposition 20.8 asserts that if one subtracts the first column of an n-by-n SCCM matrix A from the other columns and takes the $(n-1)$-by-$(n-1)$ matrix B obtained on dropping the first row and first column from the result, then one obtains a "nonsingular matrix whose nonzero eigenvalues agree with those of A." The "proof" of this eyebrow-raising result falls short.

Part IV would have been the logical place to have inserted the Perron–Frobenius theorem and its consequences for compartmental systems. One immediate consequence is that the dominant eigenvalue of a strongly connected compartmental system is real (nonpositive, in fact) and simple, and the other eigenvalues have strictly smaller real part. In particular, the system is stable in the sense of Liapunov. Furthermore, the eigenspace of this dominant eigenvalue is spanned by a positive vector. In fairness to the authors, the Perron–Frobenius theorem is also not cited in another recent book on compartmental systems by Jacquez [1].

Another prominent omission is a discussion of compartmental residence times (see [1]). This is curious since the authors do include a discussion of the expected time in a nonabsorbing state in the case of an absorbing Markov chain.

Though this book may be rather too strenuous to be used as an undergraduate text on the subject at most institutions, it should be viewed as a good mathematical introduction to the subject.

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Hal L. Smith
University of Minnesota

Erdös on Graphs: His Legacy of Unsolved Problems. *By Fan Chung and Ron Graham.* A K Peters, Natick, MA, 1999. $25.00. xiii+142 pp., softcover, ISBN 1-56881-111-X.

When I spoke to a nonmathematical friend of a book about Erdös's "Legacy of Unsolved Problems," she asked, "Isn't it rather cruel to commemorate him with a list of his failures?"

I could have made the immediate reply that the book is largely a record of Erdös's successes, or as many of them as it has room for. But for mathematicians, unsolved problems are challenges, not failures, and that is how they are presented in this book.

The problems here share a family resemblance. One of them was once popular in the following form: Six people are in a room. Show that either three of them know each other or three of them are mutually strangers. In the language of graph theory we can ask, "If the edges of a K_6 are colored red and blue, show that there is either a red or a blue triangle." Another way of putting it goes thus: "If a graph has six vertices, show that either it or its complementary graph has a triangle."

From this problem we generalize to more difficult "Ramsey-type" problems. For example, how many vertices must a graph have to ensure that either it or its complement contains a complete r-graph? Or to ensure that either it contains a complete r-graph, or its complement a complete s-graph? Typically an exact answer is given only for a few very small values of r, or of r and s. But it is shown that the minimum number n of vertices required is greater than some function f_1 of r (or of r and s) and less than some other such function f_2. We are then told how this first solution has since been improved upon by Erdös and his partners, or by other mathematicians. But usually the problem of the exact minimum value of n remains as part of Erdös's legacy of unsolved problems. It challenges his successors to produce at least a closer pair of upper and lower bounds.

I often look back on my own research and try to unify it, maintaining that each problem after the first arose out of a previous one. Having studied dissections of figures into unequal squares, one goes on to dissections into unequal equilateral triangles, or into unequal isosceles right-angled triangles. If one of these studies involves a function of graphs satisfying a recursion formula, one tries to find all the graph-functions satisfying that formula. And so on.

I suppose that other mathematicians can correlate their research problems in the same way. Certainly the correlation is clear enough with the problems of this book. One can hardly study Ramsey-type problems without becoming interested in those of Turan type. These ask how many edges an n-vertex graph must have in order to ensure the existence of a triangle—or a complete r-graph, or a copy of a given graph. Then there can be extensions to k-colored graphs. For example, how many edges must such a graph have to ensure the existence of a monochromatic triangle, or of r such triangles? So we wend our way through the book, exploring a labyrinth of interconnected puzzles in extremal graph theory.

Any researcher on finite graphs is tempted to generalize his initial results to infinite graphs, to matroids, or to hypergraphs. This book does not deal with matroids, but it shows Erdös making the other two generalizations. A hypergraph being a family of subsets of a given set, many of the hypergraphic problems are stated in the language of set theory.

When a researcher is familiar with a particular kind of finite graph he naturally asks how many of these graphs there are with n vertices or with m edges. In this book we find answers to some questions of this kind, usually in the form of a brace of upper and lower bounds. Often they are in the terminology of random graph theory. By the use of such theorems we are given a proof of Erdös's startling result that there are graphs with arbitrarily high girth and arbitrarily high chromatic number.

There is an excellent final chapter "as told by Andy Vázsonyi" about Erdös as a person. I recommend the book to all who have an interest in Erdös, his life and work, and to all who are interested in how mathematical research gets done.

W. T. TUTTE
University of Waterloo

Analyzing Multiscale Phenomena Using Singular Perturbation Methods: American Mathematical Society Short Course, January 5–6, 1998, Baltimore, Maryland.
By Jane Cronin and Robert E. O'Malley, Jr., eds.
AMS, Providence, RI, 1999. \$48.95. Softcover. ISBN 0-8218-0929-6.

It is somewhat surprising that after 30 years of great activity in singular perturbations, the field is more alive than ever, bristling with both new methods and techniques and deep mathematical questions. Also—and this is a sure sign of maturity—not only does the field draw upon other mathematical disciplines, but it is actually producing new insights and developments in subjects as different as numerical analysis, dynamical systems theory, and mathematical biology. The present volume, which consists of six papers, contains an unusually nice introduction to the basic ideas, to advanced topics, and to the relationship of asymptotics to other subjects.

The adjective "multiscale" in the title refers to the idea that in singular perturbation problems a small parameter ε induces us to use local, blown-up variables like x/ε or stretched, long-time variables like εt. This idea has turned out to be very effective, and it is described extensively in the two introductory papers, one by R. E. O'Malley, Jr., on figuring out what singular perturbations are, and one by Mark H. Holmes on the method of multiple scales. In O'Malley's paper we find a number of intriguing examples demonstrating the use of local (boundary layer) variables and a clear exposition of Tikhonov–Levinson theory. The last topic is important in analyzing initial value problems with fast transitions in time; it is also linked to geometric singular perturbation theory, as invariant manifolds play an important part in these transitions. An extensive introduction can be found in O'Malley's 1991 book [4].

The method of multiple scales (or timescales) in problems involving long-time behavior is closely related to the fundamental normalization method of averaging; this relation, including mathematical proofs of validity, has been clarified by Perko [5], a reference unfortunately lacking in this book. The method of multiple timing itself was invented in the period 1932–1935 by Krylov and Bogoliubov [2] of the Kiev school of mathematics. One of the foundations of this method is the use of secularity conditions, which were developed in the 18th century by Clairaut, Lagrange, and Laplace. Poincaré turned secularity conditions into periodicity conditions to develop his famous small parameter method for periodic solutions. The reader interested in these historical notions may consult two appendices in Sanders and Verhulst [6], which describe the history of averaging and the method of multiple timing.

On one of my visits to the Kiev Mathematical Institute during the Soviet period I attended a session chaired by its director at that time, academician Yu. A. Mitropolsky. I put forward the question of why, after the method of multiple time-scales was developed by Krylov and Bogoliubov in the 1930s, there had been no follow-up in this direction in Kiev, except in the paper by Kuzmak [3]. I also pointed out that of late the method had been developed independently in the United States and had become increasingly popular among engineers. The answer is interesting as it shows how personal taste and judgment influences the development of science: my colleagues at the Institute declared unanimously that they did not like the method and thought it was no good for studying quantitative and qualitative questions.

The introductory paper in this volume, by Holmes, has not got the correct historical facts but compensates for this by nice discussions and examples, among which is the discussion of difference equations. I was amazed, however, by the poor connection made with the current literature. Apart from the already mentioned omission of Perko's paper [5], it is stated on p. 31 that "One gap in the literature is an extension of the above results to longer time intervals, such as $0 \leq t \leq T/\varepsilon^2$." Such results were obtained in a very general form by A. H. P. van der Burgh (for references, see Sanders and Verhulst [6]).

When two branches of mathematics meet, the results are often interesting. Two-point boundary problems with turning points are hard to handle numerically, but Adjerid, Aiffa, and Flaherty here show how computational methods can use singular perturbation techniques to solve rather nasty problems. In these and other problems, large errors arise from small regions, and the authors use a quadrature approach with adaptive methods (adjusting computational meshes and accuracies) for convection-diffusion and reaction-diffusion systems. I was impressed by the examples presented.

The last decade has seen an enormous growth of dynamical systems theory and applications. In this respect the link with singular perturbation theory is very interesting, as geometric ideas complement our matched asymptotic expansion calculations by providing us with a picture of what is happening and by guiding our intuition in making calculations. I have found geometric singular perturbation theory somewhat difficult to present to students because of the subtleties of normally hyperbolic invariant manifolds, Fenichel normal forms, and the tracking of special solutions. One good introductory text is Jones [1], which gives an impression of the strength of the theory and stimulating examples. In the book under review, a nearly 50-page introduction is given by Tasso J. Kaper. This may serve well as an additional text for lectures. Kaper takes great care in pointing out the relationship between geometric and analytic (asymptotic) ideas, with fair reference to the vast literature. A number of relatively simple examples, supplementing Jones [1], do illustrate the ideas.

The book ends with two rather different topics. In a short but interesting paper Jane Cronin discusses a model of the cardiac Purkinje fiber, which plays an essential part in impulse transmission in the mammalian heart. The analysis of periodic solutions is related to the treatment of the singularly perturbed van der Pol equation, where Poincaré's expansion method also plays a part. Michael J. Ward discusses exponential asymptotics, which is concerned with boundary layer problems where straightforward application of matched asymptotic expansions leaves the problems with undetermined constants. This arises, for instance, in some linear turning point problems and in certain problems with shock-type or spike-type layers. In this treatment, the asymptotic analysis is supplemented by a projection method that produces exponentially small eigenvalues and that permits imposing solvability conditions to eliminate the indeterminacy. The reader gets a good impression of what is going on in this field, but to grasp all technical details, background reading is necessary.

These lecture notes are important to the applied mathematics community. Most of the lectures can be used in class, some will be restricted to more specialized seminars, but in all cases the intriguing mathematics and the wide applicability of the ideas will be a great stimulant for both lecturers and students.

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Ferdinand Verhulst
University of Utrecht

Understanding Search Engines: Mathematical Modeling and Text Retrieval. *By Michael W. Berry and Murray Browne.* SIAM, Philadelphia, 1999. $32.00. xiv+116 pp., softcover. Software, Environment, and Tools. Vol. 8. ISBN 0-89871-437-0.

Here is a small, compact book, written in a refreshingly informal manner, about the techniques of text searching. It is not about information retrieval so much as how Web search engines work. Prominent mention is made of a project-oriented course, "Data and Information Management," that the authors gave at the University of Tennessee in 1997, in which students designed and built their own search engines.

The first two chapters are an easy, informative read; I would strongly recommend them to people who want to learn about the issues involved in large-scale text searching without getting bogged down in details. We begin by learning about practical problems of data cleaning and about the standard vector-space representation of documents. A lot of issues in information retrieval are covered: query representation, ranking, relevance feedback, and user interfaces to query engines. The second chapter takes a closer look at data representation. There is an informal introduction to HTML and text formatting issues. Manual indexing, Yahoo-style, and automatic indexing are discussed. Then comes the process of term extraction, including normalization, stemming, and stopwords. The chapter ends with a quick look at inverted file structures and signature files. One can quibble with some of the details—for example, Huffman coding is mentioned on p. 29 in a very strange context—but the overall effect is excellent.

Chapters 3 and 4 are on vector space models, and here things change character rather dramatically. Suddenly we are inundated with linear models, matrices, and equations. The change begins with an ominous "recall that the Euclidean vector norm. . . " on p. 32, and things very rapidly get far more technical. There is an explanation of the cosine rule for measuring document similarity, formula-laden discussions of different term weighting schemes, and a description of standard storage techniques for sparse matrices. Chapter 4 begins with matrix factorization and works up to the singular value decomposition and semidiscrete representations. I found the descriptions quite comprehensible and interesting, but I felt they were badly out of character with the tone that had been set earlier for the book. And the book didn't really answer the question that I wanted to ask: How practical are these techniques for large-scale document collections? We learn on p. 56 that for very large databases the number of dimensions used may be between 100 and 300, but this is far less than the number of different words in any document collection, even when stopwords are discounted and words are stemmed. I found no other discussion of the question of scale.

The remaining chapters return to the level set at the beginning. Chapter 5 describes different types of query, Chapter 6 describes performance evaluation via recall and precision and the use of relevance feedback, and Chapter 7 gives some rudimentary information on user interfaces for searching. Chapter 8 describes the course project on which the book is based and includes full-page figures showing the interfaces the students created; although I found this discussion mildly interesting, it is certainly not central to the topic of the book.

I really like the idea of an informal, low-tech, untheoretical book about searching and search engines, and to begin with I thought this was it. Unfortunately, the overall effect was spoiled, at least for me, by including too much on latent semantic indexing (which is a research topic of one of the authors) and the vector space models on which it is based. The final giveaway was the index: among the very few terms whose entries point to more than four different pages are latent semantic indexing, rank reduction, semidiscrete decomposition, singular value decomposition, and vector space models. And the material on the student course hardly seems relevant to most readers. The authors have only just failed to write an extremely useful book.

IAN H. WITTEN
University of Waikato