1. Ordenako Ekuazio Diferentzial Linealak. Ariketen emaitzak

1.-

$$x(t) = (\int_0^t \cos(s)\, e^{-\int_0^s dr}\, ds)\, e^{\int_0^t ds} = (\int_0^t \cos(s)\, e^{-s}\, ds) e^t =$$
$$= \frac{1}{2}[(\sin t - \cos t)\, e^t + 1]\, e^t = \frac{1}{2}\,(\sin t - \cos t + e^t)$$

2.-

$$x(t) = [\int_1^t s^2\, e^{-\int_1^s \frac{1}{r}\, dr}\, ds + 3\,]\, e^{\int_1^t \frac{1}{s}\, ds} = (\int_1^t s\, ds + 3)\, t =$$
$$= (\frac{t^2}{2} - \frac{1}{2} + 3)\, t = \frac{t^3}{2} + \frac{5}{2}\, t$$

3.-(a)

$$x(t) = [\int_1^t \ln(s)\, e^{-\int_1^s -\frac{1}{r}\, dr}\, ds + C\,]\, e^{\int_1^t -\frac{1}{s}\, ds} = (\int_1^t s\, \ln(s)\, ds + C)\, \frac{1}{t} =$$
$$= (\frac{t^2}{2}\, \ln(t) - \frac{t^2}{4} + \frac{1}{4} + C)\, \frac{1}{t} = \frac{t}{2}\, \ln(t) - \frac{t}{4} + \frac{C}{t}$$

3.-(b)

$$x(t) = [\int_1^t \frac{1}{s}\, e^{-\int_1^s \frac{1}{r}\, dr}\, ds + C\,]\, e^{\int_1^t \frac{1}{s}\, ds} = (\int_1^t \frac{1}{s^2}\, ds + C)\, t =$$
$$= (\frac{t-1}{t} + C)\, t = C\, t - 1$$

3.-(c)

$$x(t) = [\int_0^t e^{-2s}\, e^{-\int_0^s -2\, dr}\, ds + C\,]\, e^{\int_0^t -2\, ds} = (\int_0^t ds + C)\, e^{-2t} = (t + C)\, e^{-2t}$$

4.- Deriba dezagun t -rekiko, bi atalak

$$\int_0^t (t-s)\, x(s)\, ds = 2t + \int_0^t x(s)\, ds, \implies \int_0^t x(s)\, ds = 2 + x(t)$$

eta berriro, t -rekiko deribatuz,

$$x(t) = x'(t) \implies x(t) = k\, e^t$$

konproba daiteke $x(t) = k\, e^t$ funtzioak hasierako ekuazioa betetzen duela $\iff$ $k = -2$, eta beraz, $x(t) = -2\, e^t$

5.- $m = \frac{1}{3}\, x(1) + 1$ eginez,

$$x(t) = [\int_0^t m\, e^{-\int_0^s \frac{1}{2}\, dr}\, ds\,]\, e^{\int_0^t \frac{1}{2}\, ds} = m(\int_0^t e^{-\frac{t}{2}}\, ds)\, e^{\frac{t}{2}} = 2m\,(e^{\frac{t}{2}} - 1)$$

$$\text{baina, } \frac{1}{3}\, x(1) + 1 = m \iff m = \frac{3}{5 - 2e^{1/2}} \text{ eta beraz,}$$

$$x(t) = \frac{6\,(e^{t/2} - 1)}{5 - 2\, e^{1/2}}$$

6.- $m = \int_0^1 s\, x(s)\, ds - x(1)$ eginez,

$$x(t) = [\int_0^t m\, e^{-\int_0^s -3\, dr}\, ds{+}1\,]\, e^{\int_0^t -3\, ds} = (m\int_0^t e^{3s}\, ds{+}1)\, e^{-3s} = \frac{m}{3} - \frac{m-3}{3}\, e^{3t}$$

$$\text{baina, } \int_0^1 s\, x(s)\, ds - x(1) = m \iff m = \frac{6\,(e^3 - 13)}{13\,(5\, e^3 - 2)} \text{ eta beraz,}$$

$$x(t) = \frac{2\,(e^3 - 13) + 63\, e^{3(1-t)}}{13\,(5\, e^3 - 2)}$$

7.- Ontzian dagoen gatz kantitatearen bilakaera aztertzerakoan lortzen dugun ekuazio diferentziala, hauxe da;

$$x'(t) = 2{,}04 - \frac{6{,}5}{60 + 2t}\, x(t) \text{ non, } x(0) = 0 \text{ den. Beraz, soluzioa,}$$

$$x(t) = [\int_0^t 2{,}04\, e^{-\int_0^s -\frac{6{,}5}{60+2r}\, dr}\, ds]\, e^{\int_0^t \frac{-6{,}5}{60+2s}\, ds} \quad \text{non}$$

$$\int_0^s \frac{6{,}5}{60 + 2r}\, dr = \ln(1 + \frac{s}{30})^{6{,}5/2} \quad \text{eta} \quad \int_0^t \frac{-6{,}5}{60 + 2s}\, ds = \ln(1 + \frac{t}{30})^{-6{,}5/2}$$

Horren ondorioz,

$$x(t) = [\int_0^t 2{,}04(1 + \frac{s}{30})^{13/4}\, ds]\, (1 + \frac{t}{30})^{-13/4} \text{ non,}$$

$$\int_0^t (1 + \frac{s}{30})^{13/4}\, ds = \frac{120}{17}\, [(1 + \frac{t}{30})^{17/4} - 1] \quad \text{den, eta orduan,}$$

$$\begin{aligned} x(t) &= \frac{2{,}04 \times 120}{17}[(1 + \frac{t}{30})^{17/4} - 1][(1 + \frac{t}{30})^{-13/4}] = \\ &= \frac{72}{5}[(1 + \frac{t}{30}) - (1 + \frac{t}{30})^{-13/4}] = \\ &= \frac{72}{5}(1 + \frac{t}{30})[1 - (1 + \frac{t}{30})^{-17/4}] \end{aligned}$$

$$v(t) = 60 + 2t = 120 \Longleftrightarrow t = 30 \text{ eta orduan eskatzen dena}$$

Sartu den gatza: $8{,}5 \times 0{,}24 \times 30 = 62{,}5$ kg, eta

atera dena: $62{,}5 - x(30) = 62{,}5 - \frac{9}{10}\,(32 - 2^{3/4}) = 33{,}9136\ldots$

8.- Olio bilakaera aztertzean lortzen den ekuazio diferentziala,

$$x\prime(t) = -\frac{1}{100+t}\,x(t) + 1 \text{ non } x(0) = 0 \text{ den. Beraz, soluzioa,}$$

$$x(t) = (\int_0^t e^{-\int_0^s -\frac{1}{100+t}\,dr}\,ds)\,e^{\int_0^t -\frac{1}{100+t}\,ds} = (\int_0^t \frac{100+s}{100}\,ds)\,\frac{100}{100+t} = \frac{t^2 + 200\,t}{200}$$

$$\text{eskatutakoa,}\quad k(340) = \frac{x(340)}{v(340)} \simeq 0{,}4742$$

9.- Ontzietan alkohol bilakaera aztertzean lortzen diren ekuazioak,

$$x_1'(t) = -\frac{1}{20}\,x_1(t) \text{ non } x_1(0) = 10 \text{ den. Eta}$$

$$x_2'(t) = \frac{1}{20}\,x_1(t) - \frac{1}{20}\,x_2(t) \text{ non } x(0) = 5 \text{ den. Beraz,}$$

$$x_1(t) = 10\,e^{-\frac{t}{20}} \quad \text{eta,}$$

$$x_2(t) = [\int_0^t \frac{e^{-\frac{s}{20}}}{2}\,e^{-\int_0^s -\frac{1}{20}\,dr}\,ds + 5\,]\,e^{\int_0^t -\frac{1}{20}\,ds} = (\int_0^t -\frac{1}{2}\,ds + 5)\,e^{-\frac{t}{20}} = e^{-\frac{t}{20}}\,(\frac{t}{2} + 5)$$

Gainera,

$$x_2'(t) = 0 \Longleftrightarrow t = 10, \text{ non } x_2(10) = 10\,e^{-\frac{1}{2}} \simeq 6{,}06531\ldots$$

$$x_1(t) = x_2(t) \Longleftrightarrow t = 10 \quad \text{non} \quad x_1(10) = x_2(10) = 10\,e^{-\frac{1}{2}}$$

$$\lim_{t\to\infty} x_1 = \lim_{t\to\infty} x_2 = 0 \text{ eta}$$