

Multiplicities of pseudoeigenvalues

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$$\begin{bmatrix} I & L \\ A & S \end{bmatrix}$$

Coimbra, July 20, 2004

Pseudospectrum of level ε

$$\varepsilon > 0, A \in \mathbb{C}^{n \times n},$$

$$\Lambda_\varepsilon(A) = \bigcup_{\substack{X \in \mathbb{C}^{n \times n} \\ \|X - A\| \leq \varepsilon}} \Lambda(X), \varepsilon\text{-pseudoeigenvalue of } A.$$

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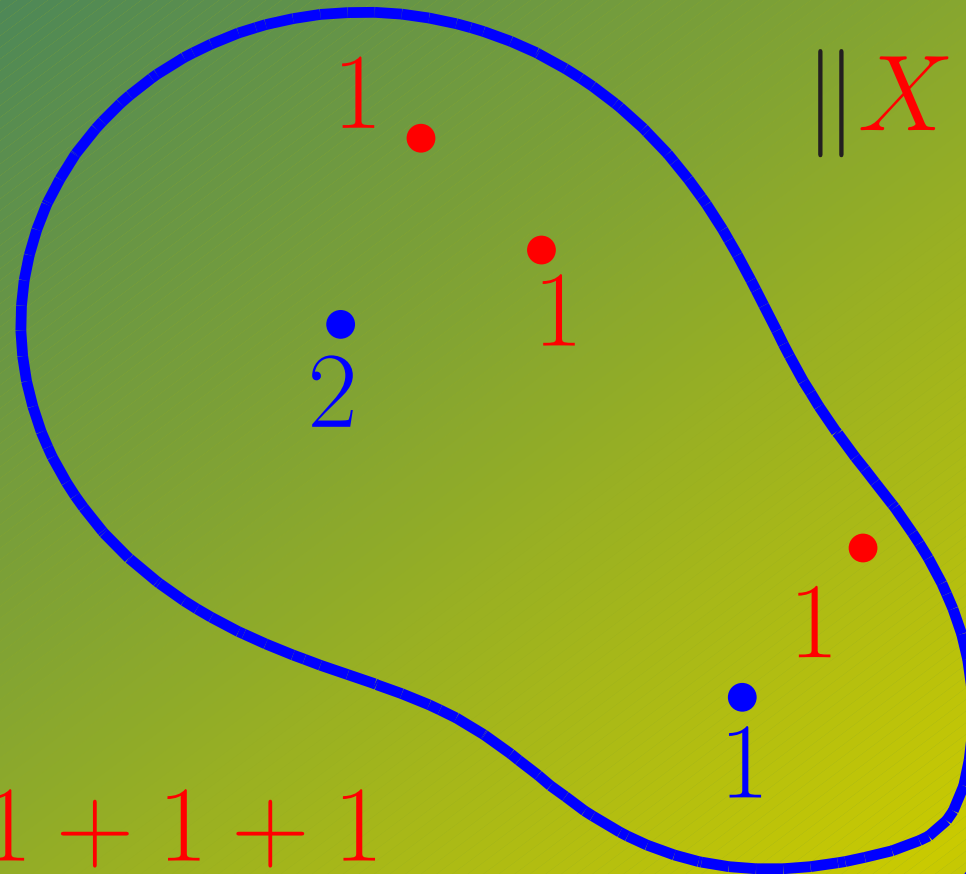
Strict pseudospectrum of level ε

$$\Lambda'_\varepsilon(A) = \bigcup_{\substack{X \in \mathbb{C}^{n \times n} \\ \|X - A\| < \varepsilon}} \Lambda(X)$$

- $\Lambda_\varepsilon(A)$ compact set; $\Lambda'_\varepsilon(A)$ open set.

Theorem 1. $A \in \mathbb{C}^{n \times n}$, $\varepsilon > 0$; $S_1, \dots, S_{\rho(\varepsilon)}$
connected components of $\Lambda'_\varepsilon(A)$; $\forall X \in \mathbb{C}^{n \times n}$,
 $\|X - A\| < \varepsilon$, $\forall i = 1, \dots, \rho(\varepsilon)$,

$$\sum_{\xi \in \Lambda(X) \cap S_i} m(\xi, X) = \sum_{\alpha \in \Lambda(A) \cap S_i} m(\alpha, A).$$



$$\|X - A\| < \varepsilon$$

$$2 + 1 = 1 + 1 + 1$$



$$1 = 1$$

Proof of Theorem 1

E top. space; continuous $\mathcal{A}: E \rightarrow \mathbb{C}^{n \times n}$;

$N(t) := \# \{ \mu \in \mathbb{C} : \det(\mu I - \mathcal{A}(t)) = 0 \}, t \in E.$

$t_0 \in E$, **bifurcation point of the spectrum** of \mathcal{A} if N is not constant on every neighbourhood of t_0 .

$$Z(t) = A + t(X - A), \quad 0 \leq t \leq 1.$$

- \exists finite $F \subset [0, 1]$,

$$\#\Lambda(Z(t)) = \begin{cases} s & \text{if } t \in [0, 1] \setminus F, \\ < s & \text{if } t \in F. \end{cases}$$

- F bifurcation points of spectrum of Z .

Lemma 2. Continuous parametrization

$$\exists \lambda_j : [0, 1] \rightarrow \mathbb{C}, j = 1, \dots, s,$$

$$\forall t \in [0, 1], \{\lambda_1(t), \dots, \lambda_s(t)\} = \Lambda(Z(t)) \subset \Lambda'_\varepsilon(A).$$

$\lambda_j([0, 1]) \subset S_i$ for only one connected component.

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Lemma 3. Constant multiplicities

$$(a, b) \subset [0, 1] \setminus F,$$

- the multiplicities of $\lambda_1(t), \dots, \lambda_s(t)$ are constant.

- Graphic explanation: [figures.pdf](#)

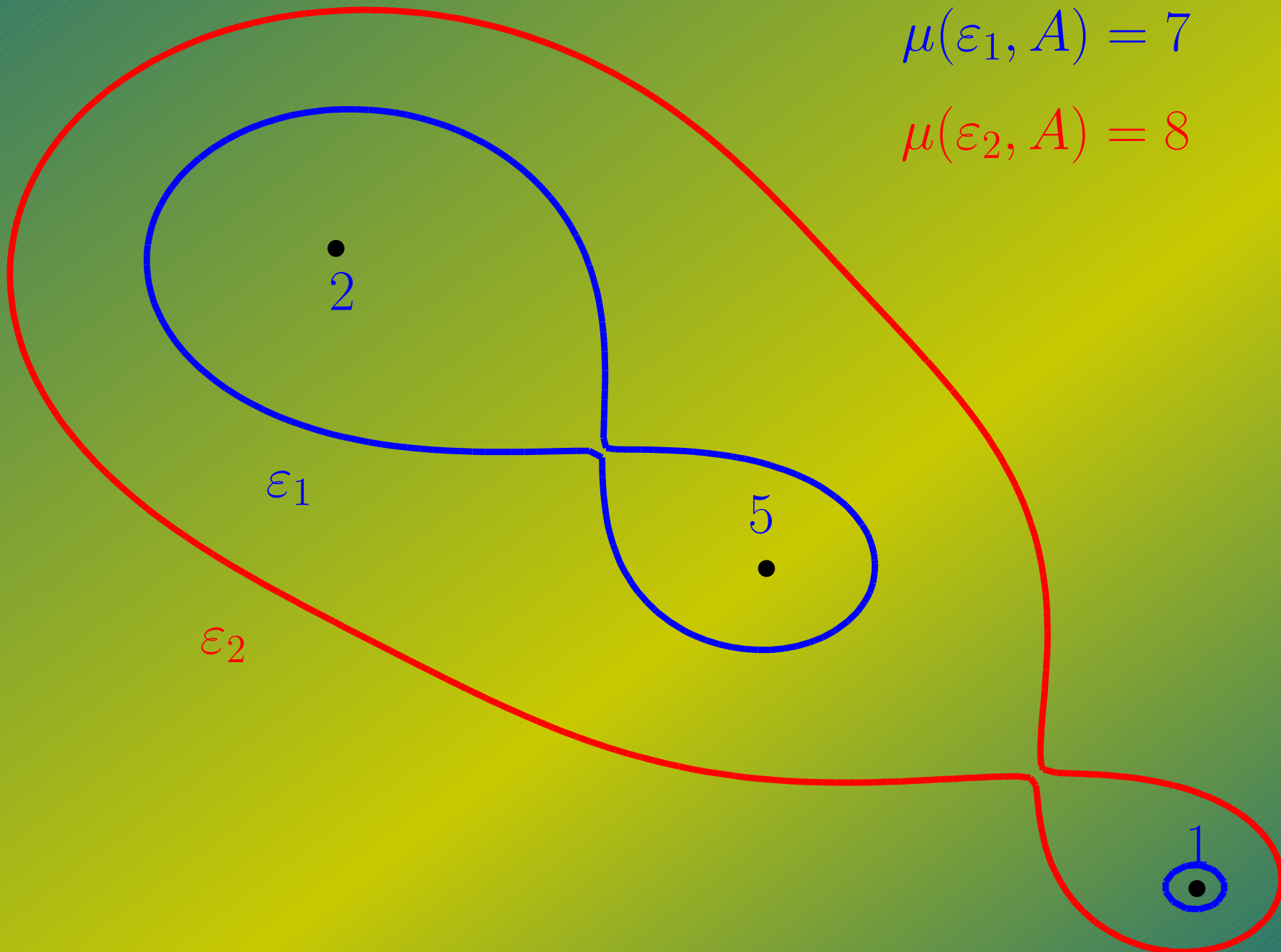
Lower bound

$$\mu(\varepsilon, A) := \max_{1 \leq i \leq \rho(\varepsilon)} \sum_{\alpha \in \Lambda(A) \cap S_i} m(\alpha, A),$$

$$m(A) := \max_{\alpha \in \Lambda(A)} m(\alpha, A), \quad m(A) \leq k \leq n,$$

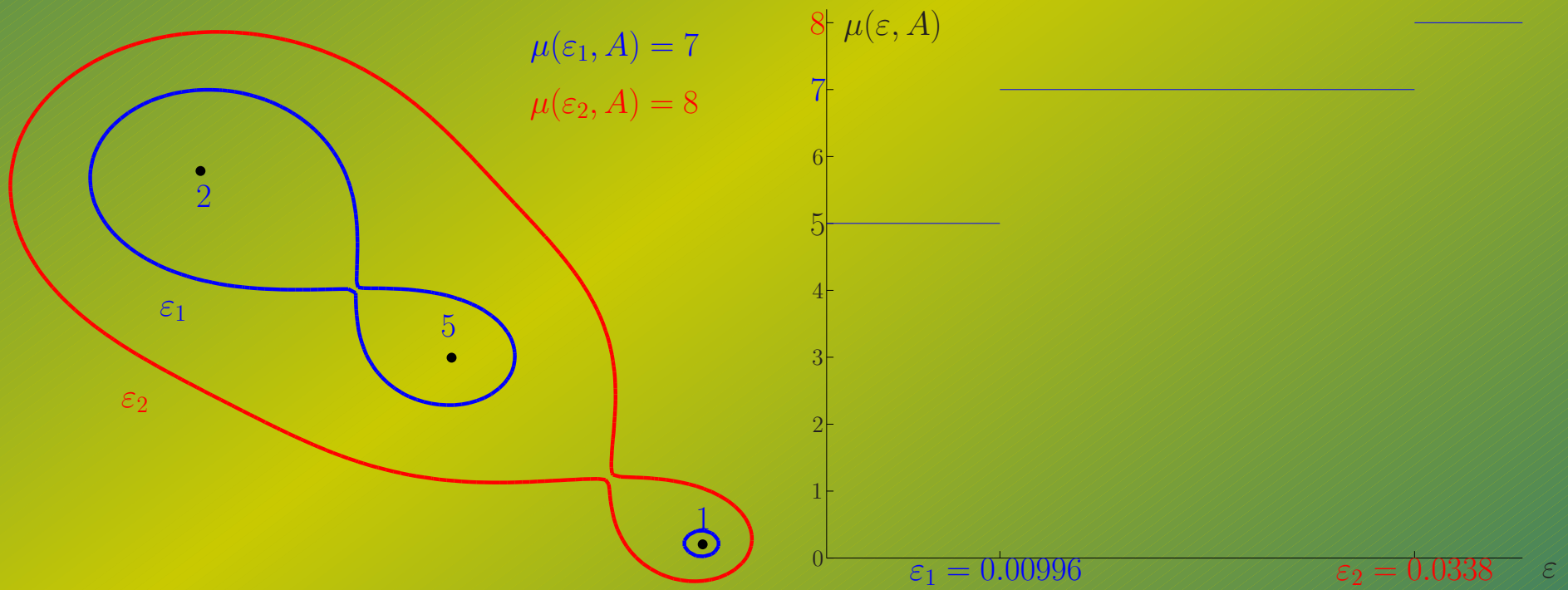


$$\sup \{ \varepsilon > 0 : \mu(\varepsilon, A) \leq k \} \leq \min_{m(X) \geq k+1} \|X - A\|.$$



$$\mu(\varepsilon_1, A) = 7$$

$$\mu(\varepsilon_2, A) = 8$$



The End

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